

MATH 195: 9/8 REVIEW WORKSHEET

Linear functions.

A *linear* function is one whose graph is a line. Algebraically, linear functions can all be written in one of these two *canonical forms*:

- **Point-slope form:** $f(x) = m(x - h) + k$, where m is the slope and (h, k) is any point on the line.
- **Slope-intercept form:** $f(x) = mx + b$, where m is the slope and b is the y -intercept.

As a special case: If the slope of $f(x)$ is 0 we also call $f(x)$ a *constant* function.

Core concepts about functions. Consider a function $f(x)$.

- The *y-intercept* or *initial value* of a function is where it crosses the y -axis. Algebraically, the initial value is $f(0)$.
- The *x-intercept(s)* or *zeros* of a function are where it crosses the x -axis. Algebraically, the zeros are the values x so that $f(x) = 0$.
- The *domain* of a function, written $\text{dom } f$, is the set of possible inputs. Unless otherwise specified assume the domain is as large as makes sense. We write domains using *interval notation*.
- The *range* of a function, written $\text{ran } f$, the set of outputs. We write ranges using *interval notation*.
- A function is *increasing* where its *slope* or *instantaneous rate of change* is positive. This means as the input moves to the right the output moves up.
- A function is *decreasing* where its *slope* or *instantaneous rate of change* is negative. This means as the input moves to the right the output moves down.

Interval notation.

An *interval* is the set of points between a minimum and maximum, possibly including or not including the endpoints.

- We write (a, b) to mean the interval of numbers x with $a < x < b$.
- To include an endpoint we use a square bracket instead of a parenthesis. For example, $[a, b)$ is the interval of numbers x with $a \leq x < b$ while $(a, b]$ is the interval of numbers x with $a < x \leq b$.
- To indicate that an interval has no minimum or no maximum we write $-\infty$ or ∞ as the endpoint. For example, $(-\infty, 0]$ is the interval of all numbers $x \leq 0$ and $[0, \infty)$ is the interval of all numbers $0 \leq x$. It never makes sense to write $[-\infty, 0]$, because $-\infty$ is not a number to be included in an interval.
- $(-\infty, \infty)$ is the interval of all real numbers, also written as $\mathbb{R}$.
- To write the *empty interval* containing no numbers we write $\emptyset$.

Practice Problems.

- (1) Find the x - and y -intercepts of the function $f(x) = 4 - 2x$.
- (2) Find the x - and y -intercepts of the function $g(x) = 3(x - 1) + 2$.
- (3) Sketch a graph of the linear function $h(x) = \frac{1}{2}(x + 1) - 1$ by finding two points on the graph and drawing the line between them. (Hint: one point is given by the form of the function, you can find another knowing the slope.)
- (4) What are the domain and range of $h(x)$ from the previous problem?
- (5) If $j(x) = mx + b$ is a generic linear function can you determine its domain and range? If so, what are they? If it depends, what does it depend on? Give as thorough an answer as possible.
- (6) Sketch a graph of a function whose range is $[-1, 3]$.
- (7) Sketch a graph of a function whose domain is $[-2, 2]$ and whose range is $[-1, 3]$.
- (8) Sketch a graph of a function whose domain is $(-\infty, \infty)$ and whose range is $[2, \infty)$.
- (9) Sketch a graph of a function whose domain is $(-\infty, \infty)$ and whose range is $[2, 4]$.
- (10) Sketch a graph of the linear function $k(x) = -x + 2$. What are its domain and range? Is it increasing or decreasing?
- (11) Consider the following four linear functions.

$$a(x) = 3x - 2$$

$$b(x) = -2x + 3$$

$$c(x) = -(x - 4) - 2$$

$$d(x) = 2(x + 1) + 2$$

Without sketching their graphs, determine which of them are increasing and which are decreasing.

- (12) Solve the equation $3x + 4 = 2$.
- (13) Find all points where the lines $f(x) = 3x + 4$ and $g(x) = 2$ intersect.
- (14) Solve the equation $2x - 1 = -x + 3$.
- (15) Find all points where the lines $h(x) = 2x - 1$ and $j(x) = -x + 3$ intersect.
- (16) The *composition* of $f(x)$ and $g(x)$ is the function you get by applying the output of one function as the input to the other: $(f \circ g)(x) = f(g(x))$.
Let $f(x) = x + 1$ and $g(x) = 2x - 1$. Confirm that $f(g(x))$ and $g(f(x))$ are two different functions, and so order matters for composition.
- (17) Sketch a graph of $g(f(x))$ from the previous problem.
- (18) Confirm that if $f(x) = mx + b$ and $g(x) = nx + c$ are two generic lines then their composition $f(g(x))$ is also a line.