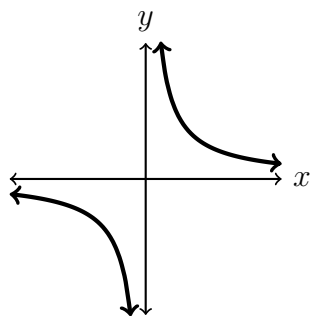


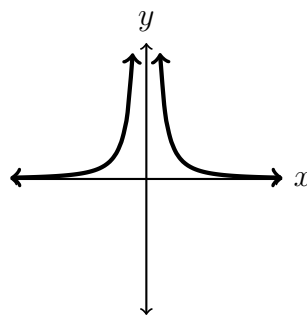
## MATH 195: 4/27 WORKSHEET

### Reciprocals of power functions.

The function  $f(x) = \frac{1}{x^n}$  looks like one of the following, based on the parity of  $n$ .



If  $n$  is odd.



If  $n$  is even.

Based on the graph one can determine the global behavior.

### Rational functions.

A *rational function* is one which can be written as a fraction of polynomials:

$$f(x) = \frac{n(x)}{d(x)}.$$

The *degree* of  $f$ , written  $\deg f$ , is the maximum of the degrees of the numerator and denominator.

- The sum, difference, product, or quotient of rational functions is a rational function.

*Zeros* of a rational function correspond to the zeroes of the numerator. *Poles* or *vertical asymptotes* of a rational function correspond to the zeroes of the denominator. If the numerator and denominator share a zero, then there is a *hole* in the graph of the rational function.

### Rational functions of degree 1.

Every rational function of degree 1

$$f(x) = \frac{ax + b}{cx + d} \quad (c \neq 0; ax + b \neq 0)$$

is a geometric transformation of  $r(x) = \frac{1}{x}$ .

- To determine the horizontal shift/location of the vertical asymptote, find the zero of the denominator.
- To determine the vertical shift/location of the horizontal asymptote, look at the ratio of the leading terms:  $\frac{ax}{cx}$ .
- To determine the orientation find an intercept and follow which side of the asymptotes it is on.

## PRACTICE PROBLEMS

- (1) Sketch a graph of the following function. Use the graph to determine its domain, range, and where it is increasing, decreasing, concave up, and concave down.

$$a(x) = \frac{-3}{(x-4)^2}$$

- (2) Do the same for the following.

$$b(x) = 2 - \frac{3}{(x+1)^3}$$

- (3) And again.

$$c(x) = 3 + \frac{2}{(2-x)^4}$$

- (4) One more time.

$$d(x) = \frac{3-x}{2x+4}$$

- (5) Find all zeroes and poles of the following rational function.

$$f(x) = \frac{(x-1)^2(x+1)}{x^2(x^2-4)}$$

- (6) Find all zeroes, poles, and holes of the following rational function.

$$g(x) = \frac{x^3(x^2+1)(x+1)}{x^2(3-x)^3}$$

- (7) Check that the following is a rational function by writing it as a single fraction of polynomials.

$$h(x) = x^2 + \frac{x+1}{x-1}$$

- (8) Check that the following is a rational function by writing it as a single fraction of polynomials.

$$j(x) = \frac{x^2/(x-1)}{(x+1)^2/(x-2)}$$

- (9) Check that the following rational function of degree 1 is a geometric transformation of  $\frac{1}{x}$  by writing it in the form  $\frac{a}{bx+c} + d$ .

$$k(x) = \frac{2x+4}{x-1}.$$

[Hint: figure out how to write the numerator as  $2(x-1) + A$ . Then,  $\frac{2(x-1)+A}{x-1}$  can be split into  $\frac{2(x-1)}{x-1} + \frac{A}{x-1}$  and you can simplify.]

- (10) Check this in general by showing how to rewrite

$$\ell(x) = \frac{ax+b}{cx+d}$$

as a geometric transformation of  $\frac{1}{x}$ . [Hint: figure out how to write the numerator as  $A(cx+d) + B$ , then follow the same steps.]