MATH 195: 4/6 WORKSHEET

Roots and their multiplicity.
A *root* or, synonymously, *zero* of a function is where its output is 0. For *continuous* functions—those without gaps or jumps—these are the only places the sign can change from positive to negative or vice versa.

For a polynomial, each root corresponds to a linear term in the complete factorization. The *multiplicity* of a root is the power of the corresponding term. For example, the polynomial

$$p(x) = x(x-1)^2(x-2)^3$$

has roots 0, 1, and 2 with multiplicities 1, 2, and 3.

Irreducible quadratic terms.
Irreducible quadratic terms are those which cannot be factored any further. They don't contribute any roots. You should make sure, however, that a quadratic really can't be factored before you discount it!

To check if a quadratic $ax^2 + bx + c$ is irreducible, calculate its *discriminant* $b^2 - 4ac$, the expression in the square root of the quadratic formula. The quadratic is irreducable if and only if its discriminant is negative.

Sign diagrams for polynomials.
If polynomial $p(x)$ has a root at $x = a$ corresponding to the term $(x-a)^m$ then the behavior of $p(x)$ near a looks like $\pm(x-a)^m$.

- Even multiplicity means the sign is the same on both sides.
- Odd multiplicity means the sign flips as you cross the root.

The end behavior of the function, determined by its leading term, tells you the sign in the two unbounded regions. You can use the rule for multiplicity of roots to then fill in the remaining regions. A *sign diagram* is a diagram which summarizes the information of where a function is positive or negative.

Sketching graphs of polynomials.
Once you know the end behavior and have a sign diagram, you can use that to sketch a graph of a polynomial. You don't have enough info to accurately place maximums, minimums, nor inflection points—you need calculus to find those—but you can mark the roots, show where the function is positive or negative, and make the end behavior clear.

Practice problems

For these polynomials, determine the leading term and the roots and their multiplicities, use that to create a sign diagram, then use the sign diagram to sketch a graph. *[Warning! Some of these are not fully factored.]*

(1) $a(x) = x(x-1)(x-2)(x-4)$
(2) $b(x) = x^2(x-1)^2(x-2)^2(x-4)^2$
(3) $c(x) = x(x-1)^2(x-2)^2(x-4)$
(4) $d(x) = (x^2-1)^2(x^2+4)x^3$
(5) $f(x) = (x^2-3x+2)(x^2-2x+3)$
(6) $g(x) = (x-1)(x+2)^2(x-3)^3(x+4)^4(x-5)^5(x+6)^6(x-7)^7(x+8)^8$
(7) $h(x) = -2x^3(x-2)(x+2)^2$
(8) $j(x) = (4x+1)(1-2x)^2(6-x)^3$
(9) $k(x) = (x^2+1)(x^2+4)(x^2+9)$
(10) $\ell(x) = 3(x-4)^3(x+2)^3(x+7)(x-1)^2$