MATH 195: 2/18 WORKSHEET

The inverse function property. If $f(x)$ and $f^{-1}(x)$ are inverses they satisfy the following properties

$$f(x) = y \Leftrightarrow f^{-1}(y) = x \qquad \text{and} \qquad f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

Applied to exponentials and logarithms, these say:

$$e^x = y \Leftrightarrow \ln y = x \qquad \text{and} \qquad e^{\ln x} = \ln(e^x) = x,$$

and similarly for other bases. These equations tell you how to rewrite equations to get a variable out of an exponent or out of a logarithm.

Rules for exponents.

A mathematically important fact about exponents is that addition inside the exponent is equivalent to multiplication outside.

- $a^{x+y} = a^x \cdot a^y$.
- $a^{x-y} = a^x / a^y$.
- $a^{xy} = (a^x)^y$.

Another important rule for exponents is the change of base formula. This formula tells us that all exponential functions are horizontal scalings of each other.

- $b^x = a^{\log_a(b)x}$.
- $b^x = e^{\ln(b)x}$.

Rules for logarithms.

Because logarithms are inverses of exponential functions, each rule for exponentials has a corresponding rule for logarithm, but with the direction swapped. For example, multiplication inside a logarithm is equivalent to addition outside.

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a(x/y) = \log_a x - \log_a y$
- $\log_a(x^y) = y \log_a x$.

There is also a change of base formula, again coming from being an inverse.

- $\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$
- $\log_b(x) = \dfrac{\ln(x)}{\ln(b)}$

Practice Problems

(1) Solve the equation for x: $0 = 2^{3x-1} - 4$.
(2) Find the x-intercept of $a(x) = 3^{4-x} - 5$.
(3) Solve the equation for x: $0 = \ln(2x + 3)$.
(4) Find the x-intercept of $b(x) = \log_{10} 4x - 5 - 1$.
(5) Solve the equation for x: $y = 3^{5x-4}$.
(6) Find a formula for the inverse of $c(x) = e^{\ln(2)x-2} + 1$.
(7) Find a formula for the inverse of $d(x) = \log_2(ex - 1) - 4$.
(8) Solve the equation $2^{2x} = 4^{3-x}$ for x.
(9) Solve the equation $3^{-2x} = 4^{x-5}$ for x. [Hint: hit both sides with a logarithm, any base, and then use rules for logarithms to simplify. Alternatively, use the change of base formula to rewrite with a common base.]
(10) Find all points where the functions $f(x) = 3^{-x}$ and $g(x) = 2^{x+1}$ intersect.
(11) Find all points where the functions $h(x) = \log_2(x)$ and $j(x) = \log_3(x - 4)$ intersect. [Hint: hit both sides with an exponential function, and use the change of base formula for logarithms.]