MATH 195: 2/11 WORKSHEET

Logarithms.

A *logarithm* is an inverse of an exponential logarithm. We write $\log_a(x)$ for the inverse of a^x, and we call a the *base* of the logarithm. Usually we are only interested in the case where $a > 1$.

- $\ln x$ is another name for $\log_e x$ and is called the *natural logarithm.*
- The meaning of $\log x$ is ambiguous; mathematicians and many programming languages will write it for the natural logarithm, computer scientists will sometimes write it for base 2, and many other sources will write it to mean base 10. For that reason, I encourage you not to write $\log x$ and to always explicitly give the base. Note that handheld calculators typically write $\log x$ for base 10.

Horizontal transformations and the order of operations.

When looking at how the algebra of a function determines what horizontal transformations were applied to its graph, everything goes backward. Adding shifts to the left, multiplying by a number > 1 shrinks the graph horizontally. If multiple horizontal transformations were applied, you also have to make sure the order of operations is backward—addition first, then multiplication.

- For example, look at $f(x) = \ln(2x - 4)$. To determine what horizontal transformations were applied, you should first rewrite it as
$$f(x) = \ln(2(x - 2)).$$
Now you can see that a shift right by 2 and a horizontal scaling by $\frac{1}{2}$ were applied.
- Another way to see this is, the horizontal shift matches where the asymptote is, and the asymptote is where the input to ln is 0. So you can solve $2x - 4 = 0$ for $x = 2$ to find the asymptote/horizontal shift.
- However you find the horizontal shift, the important thing to look at with multiplication is the sign. Positive means no horizontal reflection, while negative means there is a horizontal reflection.

Graphing and global behavior problems

Unless otherwise stated, attempt the graphing by hand first, and only use a graphing calculator to check your work.

(1) Suppose you are considering two logarithms $\log_a x$ and $\log_b x$ with different bases. Which one grows faster, the one with the smaller base or the larger base? Use a graphing calculator to confirm your answer.
(2) Like exponential functions, logarithms' graphs don't have symmetry. Give formulas for the four orientations for relections of $\ln x$ (namely, no reflection, just horizontal reflection, just vertical reflection, and reflection in both directions) and plot their graphs. For each of them, what are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(3) Sketch a graph of $a(x) = \log_2(-(x-1))$, identifying its asymptote. What are its domain and range?
(4) Sketch a graph of $b(x) = -\ln(2x+4)$, identifying its asymptote. What are its domain and range?
(5) Last time you explained why a vertical scaling of an exponential function is the same as a horizontal shift. Use this fact to explain why the horizontal scaling of a logarithm is the same as a vertical shift.
(6) Last time you explained why a horizontal scaling of an exponential function is the same as a change of base. Use this fact to explain why the vertical scaling of a logarithm is the same as a change of base.
(7) Conclude by explaining why when sketching a graph of a logarithm the only geometric transformations you have to worry about are horizontal or vertical reflections and horizontal shifts.
(8) Sketch a graph of $c(x) = -3\log_8(2x-4) + 10$. What are its domain and range? Is it increasing or decreasing? Concave up or concave down?