MATH 195: 2/9 WORKSHEET

Exponential functions.

An *exponential* function is one of the form

$$f(x) = a^x,$$

where $a \neq 1$ is a positive number which is the *base* of the function.

The constant e.

The most important exponential function, so important it is called *the* exponential function, is

$$\exp(x) = e^x.$$

Here, $e \approx 2.718281828\ldots$ is an irrational number. Here are two equivalent ways to define it:

- $e = \dfrac{1}{0!} + \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \cdots$, where $n! = n(n-1)(n-2)\cdots(2)(1)$.
- e is the value asymtotically approached by $\left(\frac{n+1}{n}\right)^n$ as $n \to \infty$.

e^x is so important because of its role in calculus. Briefly, exponential functions grow so fast that their rate of change (i.e. its slope) itself forms an exponential function. Compare to power functions, where the rate of change of x^n is nx^{n-1}, a lower power. e is the unique base which makes it so that the rate of change of e^x is e^x itself.

Graphing and global behavior problems

Unless otherwise stated, attempt the graphing by hand first, and only use a graphing calculator to check your work.

(1) Unlike most of the other functions we've previously looked at, exponential functions have no symmetry in their graph. Give formulas for the four orientations for relections of e^x (namely, no reflection, just horizontal reflection, just vertical reflection, and reflection in both directions) and plot their graphs. For each of them, is it increasing or decreasing? Concave up or concave down?
(2) Do the same for the function $\sqrt{x}$, whose graph also has no symmetry.
(3) Use a graphing calculator to graph 2^{-x} and $(\frac{1}{2})^x$. Do the same for 3^{-x} and $(\frac{1}{3})^x$. Do the same for π^{-x} and $(\frac{1}{\pi})^x$. Explain what you see.
(4) If $0 < a < 1$ what does the graph of a^x look like? Confirm your answer by checking with a graphing calculator.
(5) Consider $f(x) = -7^x - 2$. Sketch a graph, identifying the asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(6) Consider $f(x) = -2^{-2x} + 1$. Sketch a graph, identifying the asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(7) Consider $f(x) = 4 \cdot 2^x$ and $g(x) = 2^{x+2}$. What transformation is applied to the mother function of 2^x to get each of them? Use a graphing calculator to graph them. Explain what you see.
(8) Explain why a vertical scaling of an exponential function is the same as a horizontal shift.
(9) Consider $f(x) = 3^{2x}$ and $g(x) = 9^x$. Use a graphing calculator to graph them. Explain what you see.
(10) Explain why a horizontal scaling of an exponential function is the same changing the base.
(11) Conclude by explaining why when sketching a graph of an exponential function the only geometric transformations you have to worry about are horizontal or vertical reflections and vertical shifts.
(12) Sketch a graph of $f(x) = \frac{1}{2}\, e^{-3(x-1)} + 2$, identifying the asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?