MATH 195: 2/2 WORKSHEET

Formulas for quadratic functions. Let $f(x) = ax^2 + bx + c$ be a quadratic function.

- The *vertex* of $f(x)$ occurs at
$$x = \frac{-b}{2a},$$
and you can plug that value in to find the y-coordinate of the vertex.
- The *zeros* of $f(x)$ are given by the quadratic formula:
$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}.$$

Practice Problems

(1) Where is the vertex of $f(x) = x^2 - 4x + 3$? Use this information to sketch a graph, and then state the domain and range of $f(x)$.

(2) Where is the vertex of $g(x) = 2x^2 - 3x - 1$? Use this information to sketch a graph, and then state the domain and range of $g(x)$.

(3) One use of quadratic functions is in physics. If an object is thrown or dropped, its height above the ground is given by a quadratic function of time:
$$y(t) = y_0 + v_0 t - \frac{g}{2}t^2,$$
where y_0 is the initial height above the ground, v_0 is the initial vertical velocity, and g is a gravitational constant. Here, v_0 being positive means the initial velocity points upward, while negative means it points downward. The value of g depends on what units you are using. If you are using meters and seconds, then $g \approx 10\ m/s^2$.

(a) An egg is dropped from 100 meters above the ground. Write a function to model its height after t seconds and use that to determine when it will hit the ground.

(b) You toss a ball upward at a speed of 12 m/s. If the ball leaves your hand at 2 m above the ground, when will the ball reach its highest point? What is the maximum height it obtains?

Power functions.

A *power function* is one of the form $f(x) = x^n$ for a positive integer n. We will sometimes use *power function* to refer more generally to any function which is a geometric transformation of some x^n.

Concavity and inflection points.

- A function is *concave up* on an interal if the graph cups upward. More precisely, concave up means that as you move to the right the slope increases.
- A function is *concave down* on an interal if the graph cups downward. More precisely, concave up means that as you move to the right the slope decreases.
- An *inflection point* is a point where a function changes concavity.

Slope and extreme points.

An *extreme point* is either a *maximum* (a point higher than the nearby points) or a *minimum* (a point lower than the nearby points). More precisely, $f(x)$ has a maximum at $x = a$ if every point x close enough to a has $f(a) > f(x)$. Respectively, a minimum is if $f(a) < f(x)$ for close enough x.

- Extreme points are where a function changes between increasing and decreasing.
- A maximum is where a function changes from increasing to decreasing.
- A minimum is where a function changes from decreasing to increasing.