MATH 195: 1/26 WORKSHEET

Quadratic functions.
   A *quadratic* function is one which can be written as a quadratic polynomial: $f(x) = ax^2 + bx + c$, where $a \neq 0$, b, and c are parameters. This is called the *standard form* of a quadratic. If you factor it you get the *factored form* $f(x) = a(x - r_1)(x - r_2)$.

Geometric Transformations.
   A *geometric transformation* is a way of moving, stretching, reflecting, or rotating a graph. Three basic transformations are useful because they correspond to algebraic operations on functions, each coming in a horizontal or vertical variety.
- **Vertical transformations** are about the output or y-values.
  - *Translation or shift.* Moving a graph up or down. Represented as $f(x) + d$.
  - *Reflection.* Flipping a graph across the x-axis. Represented as $-f(x)$.
  - *Scale or Stretch or Dilate.* Stretching a graph vertically, or making it smaller. Represented as $c \cdot f(x)$.
- **Horizontal transformations** are about the input or x-values.
  - *Translation or shift.* Moving a graph left or right. Represented as $f(x - b)$.
  - *Reflection.* Flipping a graph across the y-axis. Represented as $f(-x)$.
  - *Scale or Stretch or Dilate.* Stretching a graph horizontally, or making it smaller. Represented as $f(x/a)$.

Vertex form for a quadratic.
   $f(x) = a(x - h)^2 + k$ has its vertex at (h, k). The sign of a tells you the orientation of the graph—does it open up or down.

Practice Problems.

(1) Find all x- and y-intercepts of $a(x) = x^2 + x - 6$.

(2) Sketch a graph of $b(x) = -2(x + 1)^2 + 3$, identifying the vertex of the parabola and the x- and y-intercepts. What are the domain and range of $b(x)$?

(3) Write $c(x) = 2x^2 + 12x + 16$ in factored form, and use this information to find the zeros of $c(x)$.

(4) Complete the square to write $d(x) = x^2 + 6x - 2$ in vertex form, and use this information to sketch a graph of $d(x)$, identifying the vertex. What are the domain and range of $d(x)$? Where is it increasing? Decreasing? Is it concave up or concave down?

(5) Determine the domain and range of $f(x) = 3(x - 3)^2 - 2$ without sketching a graph first. Where is $f(x)$ increasing? Decreasing? Is it concave up or concave down?

(6) Complete the square to write $g(x) = -x^2 + 5x + 3$ in vertex form, and use this information to sketch a graph of $g(x)$.

(7) Complete the square to write $h(x) = 2x^2 + 6x + 10$ in vertex form. Use this information to find the zeros of $h(x)$.

(8) Complete a square for the generic quadratic function $q(x) = ax^2 + bx + c$ and use this to get a formula for the zeros of $q(x)$.