MATH 195: 1/21 WORKSHEET

Linear functions.

A *linear* function is one whose graph is a line. Algebraically, linear functions can all be written in one of these two *canonical forms*:

- **Point-slope form**: $f(x) = m(x - h) + k$, where m is the slope and (h, k) is any point on the line.
- **Slope-intercept form**: $f(x) = mx + b$, where m is the slope and b is the y-intercept.

As a special case: If the slope of $f(x)$ is 0 we also call $f(x)$ a *constant* function.

Core concepts about functions. Consider a function $f(x)$.

- The *y-intercept* or *initial value* of a function is where it crosses the y-axis. Algebraically, the intial value is $f(0)$.
- The *x-intercept(s)* or *zeros* of a function are where it crosses the x-axis. Algebraically, the zeros are the values x so that $f(x) = 0$.
- The *domain* of a function, written $\operatorname{dom} f$, is the set of possible inputs. Unless otherwise specified assume the domain is as large as makes sense. We write domains using *interval notation.*
- The *range* of a function, written $\operatorname{ran} f$, the set of outputs. We write ranges using *interval notation.*
- A function is *increasing* where its *slope* or *instantaneous rate of change* is positive. This means as the input moves to the right the output moves up.
- A function is *decreasing* where its *slope* or *instantaneous rate of change* is negative. This means as the input moves to the right the output moves down.

Interval notation.

An *interval* is the set of points between a minimum and maximum, possibly including or not including the endpoints.

- We write (a, b) to mean the interval of numbers x with $a < x < b$.
- To include an endpoint we use a square bracket instead of a parenthesis. For example, $[a, b)$ is the interval of numbers x with $a \leq x < b$ while $(a, b]$ is the interval of numbers x with $a < x \leq b$.
- To indicate that that an interval has no minimum or no maximum we write $-\infty$ or ∞ as the endpoint. For example, $(-\infty, 0]$ is the interval of all numbers $x \leq 0$ and $[0, \infty)$ is the interval of all numbers $0 \leq x$. It never makes sense to write $[-\infty, 0]$, because $-\infty$ is not a number to be included in an interval.
- $(-\infty, \infty)$ is the interval of all real numbers, also written as $\mathbb{R}$.
- To write the *empty interval* containing no numbers we write $\emptyset$.

Practice Problems.

(1) Find the x- and y-intercepts of the function $f(x) = 4 - 2x$.

(2) Find the x- and y-intercepts of the function $g(x) = 3(x-1) + 2$.

(3) Sketch a graph of the linear function $h(x) = \frac{1}{2}(x+1) - 1$ by finding two points on the graph and drawing the line between them. (Hint: one point is given by the form of the function, you can find another knowing the slope.)

(4) What are the domain and range of $h(x)$ from the previous problem?

(5) If $j(x) = mx + b$ is a generic linear function can you determine its domain and range? If so, what are they? If it depends, what does it depend on? Give as thorough an answer as possible.

(6) Sketch a graph of a function whose range is $[-1, 3]$.

(7) Sketch a graph of a function whose domain is $[-2, 2]$ and whose range is $[-1, 3]$.

(8) Sketch a graph of a function whose domain is $(-\infty, \infty)$ and whose range is $[2, \infty)$.

(9) Sketch a graph of a function whose domain is $(-\infty, \infty)$ and whose range is $[2, 4]$.

(10) Sketch a graph of the linear function $k(x) = -x + 2$. What are its domain and range? Is it increasing or decreasing?

(11) Consider the following four linear functions.

$$\begin{aligned} a(x) &= 3x - 2 \\ b(x) &= -2x + 3 \\ c(x) &= -(x-4) - 2 \\ d(x) &= 2(x+1) + 2 \end{aligned}$$

Without sketching their graphs, determine which of them are increasing and which are decreasing.

(12) Solve the equation $3x + 4 = 2$.

(13) Find all points where the lines $f(x) = 3x + 4$ and $g(x) = 2$ intersect.

(14) Solve the equation $2x - 1 = -x + 3$.

(15) Find all points where the lines $h(x) = 2x - 1$ and $j(x) = -x + 3$ intersect.

(16) The *composition* of $f(x)$ and $g(x)$ is the function you get by applying the output of one function as the input to the other: $(f \circ g)(x) = f(g(x))$.

Let $f(x) = x + 1$ and $g(x) = 2x - 1$. Confirm that $f(g(x))$ and $g(f(x))$ are two different functions, and so order matters for composition.

(17) Sketch a graph of $g(f(x))$ from the previous problem.

(18) Confirm that if $f(x) = mx + b$ and $g(x) = nx + c$ are two generic lines then their composition $f(g(x))$ is also a line.