

MATH 195: TRIG REFERENCE SHEET 2

Inverse trig functions.

Because the trig functions are periodic, they are not one-to-one. To get inverse functions we have to restrict the domain. The following choices are the standard convention.

- (arcsin) Inputs are y -coordinates from -1 to 1 , outputs are angles on the right half of the circle $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
- (arccos) Inputs are x -coordinates from -1 to 1 , outputs are angles on the top half of the circle $[0, \pi]$
- (arcsin) Inputs are slopes, outputs are angles on the right half of the circle $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Finding other angles with the same output.

Using an arc trig function will give one angle θ which gives that output. There is almost always a second angle in one pass around the circle.

- (sin) The other angle is found by doing a horizontal reflection. Algebraically, $\pi - \theta$.
- (cos) The other angle is found by doing a vertical reflection. Algebraically, $-\theta$.
- (tan) The other angle is found by rotating a half-circle. Algebraically, $\pi + \theta$.

If the angle is in a cardinal direction then doing a reflection may not give a new angle. In all other cases, there are two different angles on one pass around the circle.

Solving simple trig equations.

Solve $\text{trig } x = a$ by the following process.

- (1) One solution is always $\text{arctrig } a$.
- (2) There is a second solution in one pass around the circle.
- (3) To get the general solution, add integer multiples of 2π . To represent this, we write “ $+2\pi k$, where k is an integer”. You also see “ $k \in \mathbb{Z}$ ” as shorthand for “ k is an integer”.

Here is how the solutions look for each trig function.

- $\sin x = a$:

$$x = \arcsin a + 2\pi k, \pi - \arcsin a + 2\pi k, \text{ where } k \in \mathbb{Z}$$

- $\cos x = a$:

$$x = \pm \arccos a + 2\pi k, \text{ where } k \in \mathbb{Z}$$

- $\tan x = a$:

$$x = \arctan a + \pi k, \text{ where } k \in \mathbb{Z}$$

Solving equations with other trig functions.

Rewrite in terms of sin, cos, or tan. If there is a sin and cos on opposite sides of the equation, you can move to one side to get an equation in tan.

What if the input isn't just x ?

If the input is some expression $f(x)$, to solve for x first solve for $f(x)$, then solve for x .

- For example, to solve $\tan(2x) = 1$ first get $2x = \frac{\pi}{4} + k\pi$, $k \in \mathbb{Z}$, then divide both sides by 2.
- To solve $\text{trig}(mx + b) = a$ for solve for $mx + b$, then move b over and divide both sides by m to solve for x .

In general you will get infinitely many solutions. Don't forget to include the $2\pi k$ part in your manipulations!

Using identities.

Sometimes you can use identities, such as the even odd identities, to make it easier to solve an equation.

- For example, to solve $\tan(-x) = \sqrt{3}$ you can instead solve $-\tan x = \sqrt{3}$.