STUDY GUIDE FOR MATH 195 MIDTERM 1

Types of questions

Expect there to be three kinds of questions.

- **Conceptual questions** ask you to explain something. These are writing questions.
- **Graphing and global behavior questions** ask you to sketch or interpret graphs, and to use graphs to determine the global behavior of functions.
- **Algebra and pointwise behavior questions** ask you to manipulate symbols to calculate things like intercepts or the formula for an inverse.

Topics to know

- Understand linear functions, quadratic functions, power functions, radical functions, exponential functions, and logarithms.
- Understand graphs of these functions.
- Understand geometric transformations. How do you determine which transformations were applied given a formula. How do you use this to sketch graphs.
- Understand how to find domains and ranges, and where a function is increasing, decreasing, concave up, and concave down.
- Understand how to find both x- and y-intercepts.
- Understand how to find the points of intersection of two functions.
- Understand inverses of functions. What is an inverse. When does $f(x)$ have an inverse function. How do you graph $f^{-1}(x)$ knowing the graph of $f(x)$. How do you find a formula for $f^{-1}(x)$ knowing a formula $f(x)$.
- Understand how to use rules for exponents and logarithms to simplify and solve equations involving exponential functions and logarithms.

Sample questions

(1) Find the intersection point of the two lines

$$y = 3x - 2 \qquad \text{and} \qquad y = -x + 1$$

(2) Find the x- and y-intercepts of $a(x) = 2x^2 - 8x + 5$.
(3) Find the vertex of $b(x) = -x^2 - 6x + 3$.
(4) Sketch a graph of $c(x) = -2(x-4)^2 + 3$, identifying its vertex. State its domain and range, and where it is increasing or decreasing. Is it concave up or concave down?
(5) Sketch a graph of $d(x) = 4x^2 + 2x + 1$, identifying its vertex. State its domain and range, and where it is increasing or decreasing. Is it concave up or concave down?
(6) Find all points where the parabolas $y = x^2 - 4$ and $y = (x-4)^2$ intersect.
(7) Sketch a graph of $f(x) = -x^4 - 3$, identifying its extreme point. State its domain and range, and where it is increasing or decreasing. Is it concave up or concave down?
(8) Sketch a graph of $g(x) = 2(x+1)^3 - 2$, identifying its inflection point. State its domain and range, and where it is concave up or concave down. Is it increasing or decreasing?
(9) Find all intercepts of $g(x) = 2(x+1)^3 - 2$.
(10) Give a formula for the inverse of $g(x) = 2(x+1)^3 - 2$.
(11) Sketch a graph of $h(x) = -\sqrt{2-x}$. What are its domain and range?
(12) Sketch a graph of $j(x) = -\sqrt[5]{3+x} - 4$. What are its domain and range? Where is it concave up or cancave down? Is it increasing or decreasing.
(13) Find all points where the functions $k(x) = x$ and $\ell(x) = \sqrt{1-x}$ intersect.
(14) Find all intercepts of $m(x) = 2\sqrt[3]{2x-4} + 4$.

(15) Give a formula for the inverse of $n(x) = -2\sqrt{2x+1} - 4$. As part of this you must explicitly give the restriction of the domain for the inverse.
(16) Give a formula for the inverse of $p(x) = \sqrt[3]{4-3x} - 3$. Explain why the domain of $p^{-1}(x)$ is $\mathbb{R}$.
(17) Sketch a graph of $q(x) = 4e^{2-x} + 1$, identifying the asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(18) Sketch a graph of $r(x) = -(\frac{1}{\pi})^{2x} + 3$, identifying the asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(19) Find all points of intersection of $s(x) = 3^x$ and $t(x) = 2^{4-x}$.
(20) Find all intercepts of $u(x) = 4 - 2^{3-x}$.
(21) Sketch a graph of $v(x) = -\ln(2-x)$, identifying its asymptote. What are the domain and range? Is it increasing or decreasing? Concave up or concave down?
(22) Find all intercepts of $w(x) = 4 - 2\log_2(6-2x)$.
(23) Find all x-intercepts of $y(x) = \log_3(x^2+5x+5)$.
(24) Explain why the domain of $z(x) = \log_{10}(x^2+1)$ is $\mathbb{R}$.