Math 195: Logarithms

Julia Williams

Cape Cod Community College

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Last time

- Last time we saw exponential functions, those of the form
$$f(x) = a^x, \quad \text{where } a \neq 1 \text{ is positive.}$$
- Today we'll look at a new kind of function, logarithms, which are closely related to exponential functions.

What is a logarithm?

- A logarithm is an inverse of an exponential function.
- Write $\log_a(x)$ for the inverse of a^x. We call a the base of the logarithm.
- Like with exponential functions, a must be positive and cannot be 1. Usually we only look at $a > 1$.

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- The textbook writes $\log x$ for $\log_{10} x$, sometimes called the common logarithm.
- **Caution!** Different sources are inconsistent about what $\log x$ means.
  - Mathematicians and most programming languages mean ln.
  - Computer scientists sometimes mean $\log_2$.
  - Other sources use it to mean $\log_{10}$.

  My advice: never write $\log x$, always explicitly write the base.

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- $\log_{10} 1000 = 3$ because $10^3 = 1000$.
- $\log_2 128 = 7$ because $2^7 = 128$.
- $\log_4 2 = \frac{1}{2}$ because $\sqrt{4} = 2$.
- $\log_3(\frac{1}{9}) = -2$ because $3^{-2} = \frac{1}{9}$.

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This is not very useful when x is not a neat power of a!

- $\ln 5 = ????$

Almost always, $\log_a x$ is an irrational number.

Graphs of logarithms, and global behavior

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$\log_a x$, where $a > 1$ has the following global behavior:

- $\text{dom}(\log_a) = (0, \infty)$
- $\text{ran}(\log_a) = \mathbb{R}$
- $\log_a$ is increasing on its entire domain.
- $\log_a$ is concave down on its entire domain.

Geometric transformations of logarithms

Like with other functions, knowing the graph of $\log_a x$ lets us understand graphs of its geometric transformations.

- For logarithms, the most important feature of the graph is the vertical asymptote. Its location is determined by the horizontal shift, if any.
- The other important feature is its orientation. Which reflections were done to the mother function?

A little bit of history

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- The invention of the logarithm was one of the most important mathematical discoveries in history.
- First published by John Napier in 1614.
- *Mirifici Logarithmorum Canonis Descriptio* contained tables of values of logarithms of trigonometric functions of different inputs.
- Logarithms were a tool to make calculations easier.

$$\log_a(xy) = \log_a x + \log_a y \qquad \log_a(x^n) = n\log_a x$$

- This use only became obsolete with the rise of the electronic digital computer.
- While they are no longer an essential tool for arithmetic, logarithms continue to have many important uses.

John Napier, 1616