MATH 195: ORAL FINAL

Questions

For the oral final you will present your solutions to four of these questions. You will choose one question and I will choose the other three. Unless otherwise said, all answers to calculations should be exact answers.

(1) Create a sign diagram for and sketch a graph of the following function.

$$r(x) = -2 \cdot \frac{x^2(x^2-1)^3}{x^2(x-4)^2(x-6)(x+4)^3}$$

(2) Find a formula for the inverse of the following function.

$$w(x) = \sqrt[3]{\log_7\left(1 + e^{\arctan(2x)}\right)}$$

(3) Sketch a graph of $y(x) = 3 - \sqrt[4]{4-2x}$ and its inverse. Find a formula for $y^{-1}(x)$ and determine its domain, range, where it is increasing, where it is decreasing, where it is concave up, and where it is concave down.

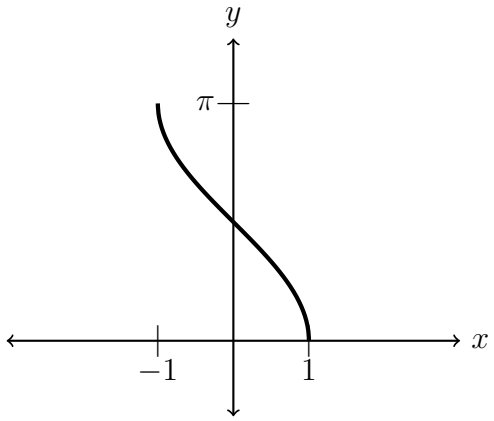
(4) Find the domain of the following function.

$$s(y) = \log_2\left(\log_2\left(y^2 - 4y + 3\right) - 3\right)$$

(5) An oracle tells you that $\sin(\pi/8) = \frac{\sqrt{2-\sqrt{2}}}{2}$. Trusting this information, find the values of the following. Give exact answers.

$$\begin{array}{llll} \sin(\pi/8), & \cos(\pi/8), & \sin(7\pi/8), & \cos(7\pi/8), \\ \sin(9\pi/8), & \cos(9\pi/8), & \sin(15\pi/8), & \cos(15\pi/8) \end{array}$$

(6) The function $c(x) = \arccos(x)$ has the following graph.



Using this graph as an example, demonstrate the six geometric transformations vertical reflection, vertical stretch, vertical shift, horizontal reflection, horizontal stretch, horizontal shift. You should give both the algebraic and geometric meanings.

(7) Find the domain of the following function. Express your answer as an interval or union of intervals, as appropriate.

$$q(u) = \sqrt{-2x(x-1)^2(x-4)^3} - \ln\left(3(x-2)^5(x+5)^2\right)$$

(8) A wave is described by the following function.

$$y(t) = 3 - 8\sin\left(\frac{\pi t}{3} - \frac{\pi}{3}\right)$$

Find the general solution for all times $t \geq 0$ where $y(t) = 7$. List the first four times this happens.

(9) It is 1956 and you are a close friend of Sergei Korolev, head of the Soviet space program which is about to launch the the first satellite into orbit. As you watch a test launch of a pair of R-7 missiles your mind wanders and you imagine their trajectories being modeled by parabolas. Imagine that the height of missile A is given by

$$a(t) = 400t - 10t^2$$

and the height of missile B, which launches 10 seconds later but with a higher initial velocity, is given by

$$b(t) = 600(t-10) - 10(t-10)^2.$$

In both cases, the quadratic functions only model the height when the output of the function is ≥ 0; at all other times the height is 0.

Determine when missile B is higher than missile A.

(10) Your friend is a hobby apiarist, and is trying to predict population growth in their bee hives. For the following, use a calculator to give decimal approximations.

(a) They attempt this with an exponential growth model:

$$B(t) = B_0 e^{rt}.$$

Knowing the initial population $B(0) = 100$ and the population after ten days $B(10) = 180$ determine the values of the parameters B_0 and r.

(b) Use this to predict the population after thirty days and how long it will take for the population to reach 1,000.

(c) According to this model, how long will it take for the population to reach 20,000?

Grading Rubric

Each question will be graded out of 25 points, for 100 points total. Grades for each problem will be assigned according to the following rubric.

- **Flawless (25 points)** Student understood the problem completely and gave a full solution with no need for outside help. When asked to justify their process they gave a cogent explanation.
- **Near flawless (23 points)** Student was able to get through a complete solution and explanation, but made small errors and needed prompting to notice and correct them.
- **Adequate (20 points)** Student required minor outside help to know how to complete their solution, or was unable to fully explain their process.
- **Borderline (15 points)** Student required substantive outside help to get all the way through the problem. Student knew the key concepts, but may have struggled to explain how they apply to this problem.
- **Needs improvement (10 points)** Student did not know how to approach the problem without outside help. Student could not adequately explain their thought process.
- **Minimal engagement (5 points)** Student did not know how to approach the problem even with outside help, but recognized some of the concepts involved.
- **No response (0 points)** Student did not show up or said nothing related to the problem.

Formula sheet

Some special values of trig functions.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\pi/6$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/3$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$

Detailed rules and format for oral final

You will schedule a 40 minute time slot for the oral final, which will be done at my office. Time slots are offered on Monday May 4th and Wednesday May 6th. You will sign up via a google sheet, which I will email to you. Please do not put off signing up for a slot; I don't want you to try to sign up last minute only to learn all the slots left conflict with your schedule.

You will present solutions to four of the questions. The first will be chosen by you, and the remainder by me. You should expect me to probe you to explain your work, and part of the grade is based on your ability to competently justify your work. I will provide paper and a writing utensil, or whiteboard if you prefer. You will be graded according to the rubric in this document. After presenting your solutions I'll let you know your grade on the final and your overall grade in the class.

The rubric mentions outside help. Here's a few things that includes: consulting your notes; using a graphing calculator or computer algebra system (using a calculator for routine arithmetic is fine); asking me for help. The formula sheet in this document does not count as outside help, and I will have a copy available for you.

Advice for preparing

- Do all questions fully in advance! Practice writing down your solutions and explaining them.
- You are allowed to talk to classmates and tutors when preparing, but make sure you understand everything yourself. If I ask you "why did you do X?" then "the math learning center said it's the right thing" isn't a good answer.
- Check your solutions. Computer tools, especially graphing calculators, are great for checking your work.
- You can stop by office hours to ask about the class material, but I will not give hints for the questions for the final.