

Math 130

Unit 1: Logic

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 - Data within a database.
 - Our moral obligations.
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- Logic is a tool for organizing our thoughts about the **states of affairs**.
 - The relationship between mathematical principles.
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 - Our moral obligations.
- Major uses in philosophy, mathematics, computer science, and linguistics.
- In this unit our focus will be **formal logic**, also called **symbolic logic**, where we express things in a formal language.
- Akin to how arithmetic sentences express facts about quantity, we can express logical facts in this language.

Arguments

One use of logic is to analyze **arguments**. The meaning is like in the phrase “legal argument”—a precise series of steps of reasoning to reach a conclusion.

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 - It is Tuesday.
 - $\therefore$ I have a Math 130 class to attend.
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 - It is Tuesday.
 - $\therefore$ I have a Math 130 class to attend.
- ($\therefore$ means “therefore” and is used to mark the conclusion of the argument.)
- If my professor hated me she’d make me do homework.
 - My professor doesn’t hate me.
 - $\therefore$ My professor won’t make me do homework.

Expressing arguments in ordinary language

- If it is Tuesday then I have a Math 130 class to attend.
- It is Tuesday.
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If it is Tuesday then I will have a Math 130 class to attend. It is Tuesday. Therefore I have a Math 130 class to attend.

It's Tuesday and I have Math 130 on Tuesdays, so I have a Math 130 class to attend.

I have a Math 130 class to attend because that class is on Tuesdays and today is Tuesday.

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In ordinary language we don't always put the conclusion at the end and we don't always separate things into different sentences.

- Conclusions are often indicated by words like so, thus, therefore, hence, accordingly.
- Premises are often indicated by words like because, since, given that.

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- Gollum threw the One Ring into the fires of Mount Doom
- $\therefore$ Frodo did not throw the One Ring into Mount Doom.
- I need a boost in energy.
- $\therefore$ I should drink coffee.

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- Gollum threw the One Ring into the fires of Mount Doom
- If Gollum threw the One Ring then Frodo did not.
- $\therefore$ Frodo did not throw the One Ring into Mount Doom.

- I need a boost in energy.
- If I need a boost in energy then I should drink coffee.
- $\therefore$ I should drink coffee.

Sentences in logic

Not every English sentence makes sense to be part of an argument.

- What's your favorite novel?
- Don't sleep through your alarm.

We are interested in **declarative** sentences, those that state such and such is the case.

Think: does it make sense to respond to the sentence with “this is true (false)” or with “I (dis)agree”?

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Warning! This does not track the fact versus opinion distinction. We can use logic to analyze the consequences of “Tori Amos’s latest album slaps” just as well as we can to analyze “a hydrogen atom has one proton”.

We aren’t interested in absolute truth, but “truth in the sense of X ”.

- True in the sense of physics.
- True in the sense of Prof. Williams’s opinions.
- True in the sense of what happens in *Lord of the Rings*.

Logic can be used to analyze all of these

Validity

We want to distinguish between these two kinds of argument.

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We want arguments where the conclusion is a **consequence** of the premises. If you agree with the premises then you must agree with the conclusion.

We call such an argument **valid**.

On the other hand, an argument is **invalid** if it's possible for the premises to be true but the conclusion to be false.

Soundness

If a valid argument moreover has that all of its premises are all true then we call it **sound**.

- If a number is prime then its square root is irrational.
- 3 is prime.
- $\therefore \sqrt{3}$ is irrational.

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Checking soundness can be very hard, and it won't be our focus! We will say that is someone else's problem.

- There's either an even number or odd number of electrons in the universe.
- There is not an odd number of electrons.
- $\therefore$ There is an even number of electrons.

A guiding question

- Is there a procedure we can use which will always allow us to distinguish a valid argument from an invalid argument?