MATH 130: 3/26 WORKSHEET
PROBABILITY: MORE ABOUT AVERAGES

Last time we talked about three notions of average in probability.

- The *mean* or *expected value* of a random variable is a weighted sum—add up the possible values weighted by their probabilities.
- The *median* of a random variable is where it's equally likely for the random variable to be on either side.
- The *mode* of a random variable is the value that occurs most often.

Today we will use a graphing calculator, such as the one found at `https://desmos.com/calculator`, to visualize the meaning of these notions for *continuous* probability distributions. These are where there are infinitely many possible outcomes, rather than the *discrete* or finite ones we've focused on. The mathematics of continuous probability distributions is more difficult, but we can understand them using computer tools.

The normal distribution.

An important continuous distribution is the *normal distribution*. In desmos, a normal distribution can be created by typing `normaldist()`. This creates the standard normal distribution, whose mean is 0.

- For continuous distributions, the area under the curve represents the probability that the value of X is in a certain region. Clicking the "Cumulative Probability" drop-down you can put in different endpoints and desmos will calculate the likelihood X falls between them. Plug in different values to get a feel for this.
- Type `normaldist(2)` to get a normal distribution with a mean of 2. How does its graph compare to the graph with a mean of 0? Try plotting normal distributions with other means to compare.
- For continuous distributions, the median is where half the area is to the left and half the area is to the right. Use the "Cumulative Probability" drop-down to confirm that the median of a normal distribution is its mean.
- For continuous distributions, the mode is the value where the graph has its maximum. Where is the mode of a normal distribution?
- The *standard deviation* of a distribution is a measure of how spread out it is. Type `normaldist(0,2)` to create a normal distribution with mean 0 and standard deviation 2. What effect does this have on the graph? What if you change the standard deviation to different values?
- You can give your distribution a name by typing like `N = normaldist(2,3)`, then you can type `mean(N)`, `median(N)`, `stdev(N)` to have desmos calculate the mean, median, and standard deviation.

The uniform distribution.

A *uniform* distribution is one where all values between a minimum and a maximum are equally likely. You can graph a uniform distribution with minimum 1 and maximum 5 by typing `U = uniformdist(1,5)`

- Looking at the graph, try to visualize where you think the mean and median should be. Confirm your guess by typing `mean(U)` and `median(U)`.
- Plot uniform distributions with different minimums and maximums and determine their means and medians. Do you notice a general pattern? In general, what are the mean and median of a uniform distribution?
- Also determine the standard deviations of your different uniform distributions by typing `stdev(U)`. What do you notice?
- Why does a uniform distribution not have a unique mode? Explain.

The binomial distribution.

You can also use desmos to visualize discrete distributions, like the *binomial distribution*. This distribution counts the number of heads if you flip a coin (possibly biased) multiple times. Type `B = binomialdist(10,0.5)` to plot a binomial distribution representing 10 coin flips with a 0.5 chance of heads.

- Look at the graph and try to visually determine the mean and median. Confirm your guess by typing `mean(B)` and `median(B)`.
- If you increase the probability of a heads to be $> 1/2$, how does that affect the graph? What does it do the mean and median? How about if you decrease the probability of heasd to be $< 1/2$?
- Change the number of flips. How does that affect the graph of the distribution?
- Graph a normal distribution with the same mean and standard deviation by typing `normaldist(mean(B),stdev(B))`. What do you notice about the two graphs? What if you change the parameters of your binomial distribution?

The Poisson distribution.

Another discrete distribution is the *Poisson distribution*. Graph a Poisson distribution with mean 4 by typing `P = poissondist(4)`.

- Confirm that the mean really is 4 by typing `mean(P)`. Type `median(P)` to see what the median is and `stdev(P)` for the standard deviation.
- What happens if you change the mean? How does that affect the graph and its statistics?
- Plot a normal distribution with the same mean and standard deviation by typing `normaldist(mean(P),stdev(P))`. What do you observe? What if you change the parameter in your poisson distribution?

Practice Problems

None for today!