MATH 130: 3/17 WORKSHEET
PROBABILITY: FUNDAMENTAL PRINCIPLES

Probability is the mathematics of quantifying uncertainty. Today we lay out the basic concepts we need to understand.

Sample spaces and events.

A *sample space* is a set of possible outcomes from a *random trial* or *experiment*. We often write a sample space as S.

- If you roll a six-sided die and look at the value the sample space is the set of whole numbers from 1 to 6.
- If you roll two six-sided die and sum their values the sample space is the set of whole numbers from 2 to 12.
- If you capture lobsters and record their weights, it makes sense to use $\mathbb{R}$, the set of real numbers, as the sample space.

For more advanced applications, sample spaces can be infinitely large. For this unit, we will focus on finite sample spaces.

A *event* is a possible result you might record from a random trial. More precisely, an event is a subset of the sample space. If an event consists of a single outcome we call it an *atomic event*. We use capital letters $E, \ldots$ for events with lowercase letters $a, \ldots$ for atomic events.

- If you roll a six-sided die and what to know what value was rolled, you are looking at atomic events.
- If you roll two six-sided die and sum their values, you are no longer looking at atomic events. The event "roll a 7" is the set of all possible ways for the two dice to sum to 7. You could roll 1 and 6 or you could roll 2 and 5.
- If you capture lobsters and record their weights, you are interested in events like "the weight is between 1.9 and 2.1 pounds". Atomic events like "the weight is exactly 2 pounds" are too unlikely to be useful.

Probability distributions

A *probability distribution* is a function which says how likely each event is. We write $P(E)$ to represent the probability the event E occurs.

- $P(\emptyset)$, the probability that nothing happens , is always 0.
- $P(S)$, the probablity that something happens, is always 1.
- For finite sample spaces, atomic events a have a probability $P(a)$, which must be between 0 and 1. Then $P(E)$ is the sum of $P(a)$ for each $a \in E$ for any event E.
- The $P(a)$'s for all atomic events must sum to 1.

A probablity distribution is *uniform* if every atomic event has the same likelihood. We will mostly focus on uniform distributions, and we say things like "choose uniformly at random" to indicate we are looking at a uniform distribution.

- For an event (or sample space), write $|E|$ for the number of outcomes in the event.
- For a uniform distribution, each atomic event a has probability
$$P(a) = \frac{1}{|S|}.$$
- For a uniform distribution, each event E has probability
$$P(E) = \frac{|E|}{|S|}.$$

Practice problems

Please write all answers as fractions.

(1) Consider the random trial where you roll two six-sided dice. The events you are interested in are the different possible sums of their values. Create a table of all possible outcomes and determine the probability of each of the eleven events.
(2) Consider the random trial where you roll a six-sided die. The events you are interested in is whether the value is at least n, for different values of n from 1 to 6. Determine the probability of each of these 6 events.
(3) Consider the random trial where you roll a twenty-sided die. The events you are interested in is whether the value is at least n, for different values of n from 1 to 20. Write a formula that gives the probability of the event "the value is at least n".
(4) You and your three roommates are splitting up household responsibilities. If you each independently pick a day of the week to do dishes, how likely is it that each of you picked a different day? (Assume that each of you picks one of seven weekdays uniformly at random.)
(5) Your ATM PIN consists of four digits 0 to 9. If you chose your PIN uniformly at random, what is the probability that no digit repeats?
(6) What is the probability that the digits alternate even-odd-even-odd? (Possibly with repeated digits.)
(7) What if your PIN is required to not have any digits repeat? If you chose uniformly at random from the non-repeating PINs, what is the probability that the digits alternate even-odd-even-odd.