MATH 130: 2/17 WORKSHEET
LOGIC: ARGUMENTS AND VALIDITY

To close out the unit on logic we will look at one last use of truth tables.

Arguments.
An *argument* is a reasoned attempt to convince someone of something. (Think legal argument, not argument as in verbal fight.) In logic, we formalize this as, an argument is a series of *premises* followed by a *conclusion*.

Not all arguments are equally convincing. Consider these two.

- (Premise) If the Patriots win the Super Bowl I will be happy.
- (Premise) The Patriots won the Super Bowl.
- (Conclusion) Therefore, I am happy.

- (Premise) If the Patriots win the Super Bowl I will be happy.
- (Premise) I am happy.
- (Conclusion) Therefore, the Patriots won the Super Bowl.

Validity and soundness.
An argument is *valid* if whenever each of its premises is true its conclusion must be true. A valid argument is moreover *sound* if each of its premises is actually true. Soundness requires work outside of logic to check, but validity we can cast using truth tables: an argument is valid if any row of the truth table where every premise is true has every conclusion is true.

Using this we can check some common argument patterns and which ones are valid.

- (*Modus ponens*) 1. If A then B. 2. A. 3. Therefore, B.
- (Affirming the consequent) 1. If A then B. 2. B. 3. Therefore, A.
- (*Modus tollens*) 1. If A then B. 2. Not B. 3. Therefore, not A.
- (Denying the antecedent) 1. If A then B. 2. Not A. 3. Therefore, not B.

The limits of propositional logic.
Here's an argument, I hope you think it's valid.

- (Premise) Every dog goes to heaven.
- (Premise) Leo is a dog.
- (Conclusion) Therefore, Leo goes to heaven.

This argument can't be formalized in propositional logic because it doesn't have a way to handle "all Xes are Y" statements. There are other, more complicated logical systems which can handle statements like this. A thorough logic textbook will cover *predicate logic* and other systems that can handle more aspects of reasoning.

Practice Problems

For each argument pattern, write the premises and conclusions as logical statements. Then use truth tables to determine which of them are valid.

(1) (Disjunctive syllogism) 1. A or B. 2. Not A. 3. Therefore, B.
(2) (Affirming a disjunct) 1. Either A or B. 2. A. 3. Therefore, not B.
(3) 1. A. 2. Therefore, A or B.
(4) (Hypothetical syllogism) 1. If A then B. 2. If B then C. 3. Therefore, if A then C.
(5) 1. A. 2. B. 3. Therefore, A and B
(6) 1. A or B. 2. A. 3. Therefore, A and B.
(7) 1. A and if A then B. 2. Therefore, B.
(8) 1. Neither A nor B. 2. Therefore, if A then B.
(9) (*Non sequitur*) 1. A. 2. Therefore B.