MATH 130: 2/12 WORKSHEET
LOGIC: USES OF TRUTH TABLES, AND EQUIVALENCE

Last week we looked at truth tables. Today we'll look at one thing truth tables are useful for.

Equivalence.
Two propositions are *equivalent* if their truth tables have the same pattern. This means that they express the same logical idea but in different ways.

Last week some of the example truth tables we looked at were secretly examples of equivalence.

- P and $\sim\sim P$ are equivalent.
- $P \to Q$ and $\sim Q \to \sim P$ and $\sim P \vee Q$ are all equivalent.
- $P \leftrightarrow Q$ and $(P \to Q) \wedge (Q \to P)$ are equivalent.
- $P \to (Q \to R)$ and $(P \wedge Q) \to R$ are equivalent.

Here are some more examples of equivalent propositions.

Order doesn't matter for And and Or.

- $P \wedge Q$ and $Q \wedge P$ are equivalent.
- $P \wedge (Q \wedge R)$ and $(P \wedge Q) \wedge R$ are equivalent.
- $P \vee Q$ and $Q \vee P$ are equivalent.
- $P \vee (Q \vee R)$ and $(P \vee Q) \vee R$ are equivalent.

Compare these to the corresponding rules for addition and multiplication. When you add or multiply, the order doesn't matter.

De Morgan's Laws.

- $\sim(P \wedge Q)$ is equivalent to $\sim P \vee \sim Q$.
- $\sim(P \vee Q)$ is equivalent to $\sim P \wedge \sim Q$.

Tautologies and contradictions.

A *tautology* is a proposition which is always true. A *contradiction* is a proposition which is always false. That is, if you make a truth table for the proposition it is either all T's for a tautology or all F's for a contradiction. A proposition which is neither a tautology nor a contradiction is called *contingent*.

- $P \vee \sim P$ is a tautology.
- $P \wedge \sim P$ is a contradiction.
- P is contingent.

Practice Problems

(1) For the following pairs of propositions, use truth tables to determine whether they are equivalent.
   (a) A and $\sim A$.
   (b) $A \vee B$ and $\sim A \to B$.
   (c) $A \to B$ and $A \leftrightarrow B$.
   (d) $A \vee \sim A$ and $A \to A$.
   (e) $A \vee (B \wedge C)$ and $(A \vee B) \wedge (A \vee C)$.

(2) For the following propositions, use truth tables to classify them as a tautology, contradiction, or contingent.
   (a) $A \to (B \to A)$.
   (b) $(A \wedge B) \vee (B \wedge A)$.
   (c) $(A \wedge B) \to A$.
   (d) $A \to (A \wedge B)$.
   (e) $\sim(A \to B \vee B \to A)$.

(3) What does the Venn diagram for a tautology look like? Draw it. What does the Venn diagram for a contradiction look like? Draw it.

(4) Explain why all tautologies are equivalent to each other. How does this make you feel?

(5) Explain why all contradictions are equivalent to each other. How does this make you feel?