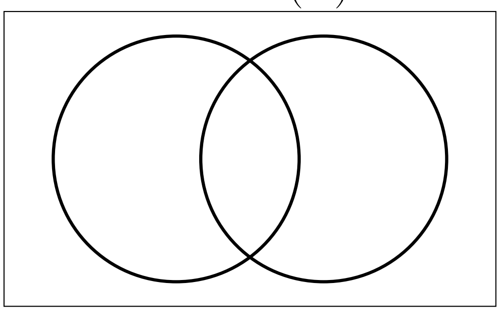
MATH 130: 2/10 WORKSHEET
LOGIC: VENN DIAGRAMS

Last time we looked at truth tables, a means of expressing the behavior of a proposition. Today we'll look at at a graphical way of expressing the same content.

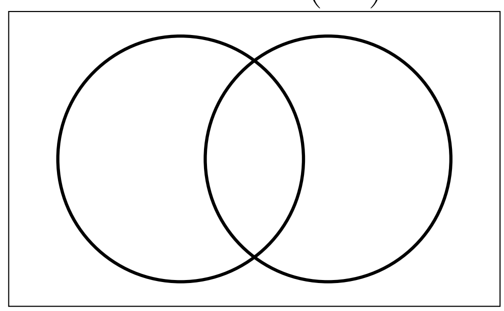
Venn diagrams.

A *Venn diagram* is a collection of overlapping circles. Each circle represents a variable, with inside meaning true and outside meaning false. Regions of the diagram correspond to different possible combination of truth values for the variables, with a region being shaded in corresponding to an output of true.

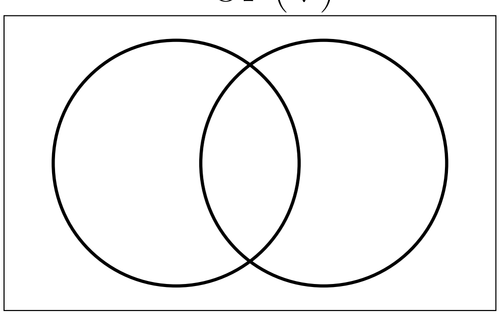
And ($\wedge$)



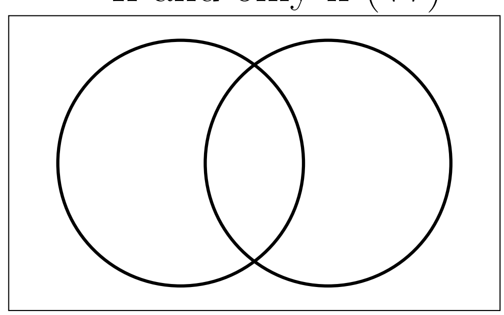
If-then ($\rightarrow$)



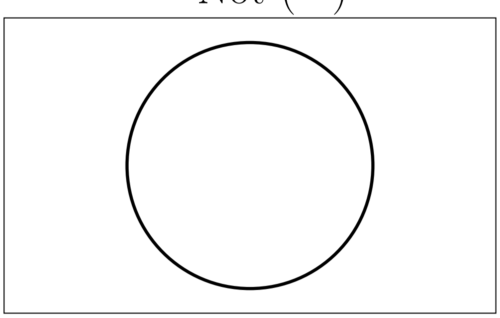
Or ($\vee$)



If and only if ($\leftrightarrow$)



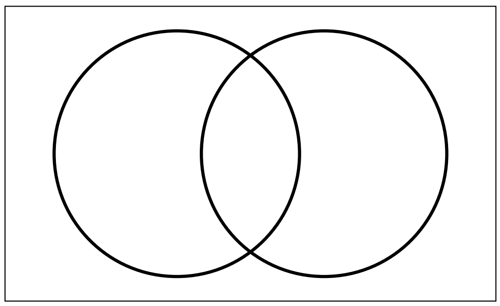
Not ($\sim$)



Defining new connectives.

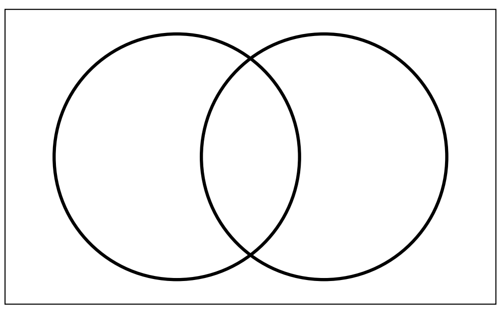
You can use Venn diagrams to describe how new connectives should behave. Let's look at two examples, and the corresponding truth tables.

Exclusive-or (xor)



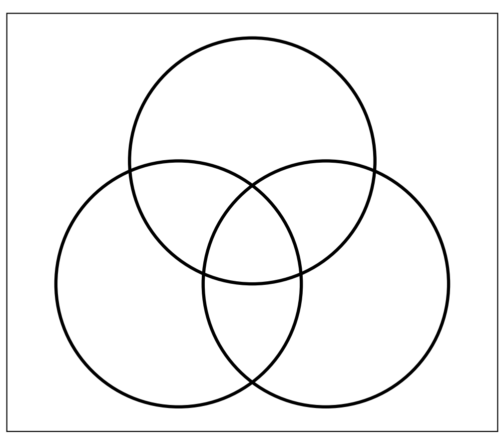
P	Q	P xor Q
T	T	
T	F	
F	T	
F	F	

Neither-nor (nor)



P	Q	P nor Q
T	T	
T	F	
F	T	
F	F	

Venn diagrams with three circles.



Practice Problems

(1) Draw Venn diagrams corresponding to the following three truth tables:

(a)

P	Q	
T	T	F
T	F	T
F	T	T
F	F	T

(b)

P	Q	
T	T	F
T	F	T
F	T	F
F	F	F

(c)

P	Q	R	
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

(2) For the following propositions, fill out truth tables for them and draw Venn diagrams for them.
   (a) $P \wedge {\sim}Q$
   (b) ${\sim}P \leftrightarrow Q$
   (c) $P \to ({\sim}Q)$
   (d) $P \vee {\sim}P$
   (e) ${\sim}(P \vee Q) \wedge P$.
   (f) $P \vee (Q \wedge R)$

(3) Which do you find easier to work with, Venn diagrams or truth tables? Explain.