MATH 130: 1/29 WORKSHEET
LOGIC: TRUTH TABLES AND THE MEANING OF CONNECTIVES

The logic we are studying, called *propositional logic* is based on the idea that the meaning of logical symbols depends only on the truth values of their inputs. That is, the truth of a statement like $A \wedge B$ doesn't depend upon the meaning of A and B, only on whether they are true or false. Thus, to express the meaning of a connective it is enough to list out its truth values for all possible combinations of its inputs.

Each variable has two possible truth values: true (T) or false (F). Thus, a *binary* connective—one with two inputs—has $2^2 = 4$ possible combinations of its inputs. In general, if you have n variables there are 2^n possible combinations of their truth values.

A *truth table* is a device to summarize this information for a complex proposition. On the left are rows for each possible combination of inputs. On the right is the truth value of the proposition for the corresponding inputs. One use of truth tables is to state the meaning of the five connectives.

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$P \to Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$P \leftrightarrow Q$
T	T	
T	F	
F	T	
F	F	

P	$\sim P$
T	
F	

You can make truth tables for more complicated propositions. To do this, you work inside out. Start by setting up the rows of your truth table. Then determine the behavior of the innermost connectives. Use that to work outward one more layer, and repeat until you have the truth value of the entire proposition.

P	Q	$\sim P \vee Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$\sim Q \to \sim P$
T	T	
T	F	
F	T	
F	F	

Practice problems

(1) Fill out a truth table for $\sim P \to Q$.
(2) Fill out a truth table for $P \vee (Q \to \sim R)$.
(3) Fill out a truth table for $(P \to Q) \wedge (Q \to P)$. Compare to the truth tables for the five connectives. What do you notice? Explain.
(4) Fill out truth tables for $\sim\sim P$, $\sim\sim\sim P$, and $\sim\sim\sim\sim P$. Do you notice a pattern? What would the truth table look like if you had n many $\sim$s?
(5) Fill out truth tables for $\sim(P \vee Q)$ and $\sim(P \wedge Q)$.
(6) Fill out a truth table for $(P \wedge \sim Q) \vee (\sim P \wedge Q)$.
(7) Fill out truth tables for $P \to (Q \to R)$ and $(P \wedge Q) \to R$. What do you notice? Explain.
(8) If a proposition had 4 variables in it, how many rows would you need for its truth table? What if it had 5 variables?