MATH 130: 1/27 WORKSHEET
LOGIC: PROPOSITIONS

A *proposition* is a statement that is *truth-apt*—it can unambiguously be true or false. We will assume a common sensical view of truth, and leave careful analysis to a philosophy class. Note: your context can determine what makes sense to treat as a proposition.

Examples.

- "Snow is white" is a proposition.
- "At midnight January 1st, 2026 (eastern time) there were an odd number of humans in the USA" is a proposition. While it is unfeasible to determine whether it's true, it's a matter of counting so there's an objective answer.
- "Frodo throws the One Ring into Mount Doom" is a statement about a work of fiction. If you're using logic to model what happens in that fiction it makes sense to treat it as a proposition.
- "Black coffee tastes good" maybe seems like an opinion, not something that is truth-apt. But if you're using logic to model to beliefs of a specific person it makes sense to treat it as a proposition.

Logicians abbreviate propositions with a single letter $A, B, C, \ldots$, much like how mathematicians will use letters for number variables.

Logical *connectives* are ways to combine simpler propositions to make a more complex proposition.

- (Atomic propositions) Variables $A, B, C, \ldots$ are the starting point.
- (And/conjunction) $A \wedge B$ is logician-speak for "A and B".
- (Or/disjunctcion) $A \vee B$ is logician-speak for "either A or B, or possibly both" or "at least one of A and B". We call this an *inclusive or*, because it allows the possibility both are true.
- (Not/negation) $\sim A$ is logician-speak for "A is not true" or "not A".
- (If-then/conditional) $A \to B$ is logician-speak for "if A, then B". There are many ways if-then statements can be written in English: "B if A", "A is a sufficient condition for B", "B is a necessary condition for A", "when A, B", etc.
- (Iff/biconditional) $A \leftrightarrow B$ is logician-speak for "A if and only if B" or "A and B are equivalent". The phrase "if and only if" is commonly abbreviated "iff". Philosophers express $A \leftrightarrow B$ as "A is a necessary and suffient condition for B".

Parentheses are used to show order of operations, like in arithmetic. You should include parentheses whenever the order is unclear. It is common convention that the order of operations goes Parentheses, Not, And/Or, If-then/Iff, but extra parentheses never hurt.

If you have two or more Ands in a row (or Ors in a row) you do not need parentheses, much like you don't need parentheses to show the order for adding three numbers. Later we will justify why order doesn't matter for And and Or.

Propositions written in normal English can be translated to logic and vice versa. For these examples, let S abbreviate "there was a snow day Monday", A abbreviate "Angola secured its independence from Portugal in 1975", and P abbreviate "plants get energy through photosynthesis". (All of these propositions happen to be true.)

Examples. Which of these complex propositions do you think are true?

- $S \wedge P$ abbreviates "there was a snow day Monday and plants get energy through photosynthesis".
- "Plants get energy through photosynthesis but there was not a snow day on Monday" can be abbreviated $P \wedge {\sim}S$.
- $A \to P$ abbreviates "if Angola won its independence from Portugal in 1975, then plants get energy through photosynthesis".
- $(A \vee S) \to {\sim}P$ abbreviates "if either Angola won its independence in 1975 or there was a snow day Monday, then plants don't get energy through photosynthesis".
- ${\sim}(P \wedge {\sim}A)$ abbreviates "it is not the case that both plants get their energy from photosynthesis and Angola did not win its independence from Portugal in 1975".

Practice Problems

(1) Translate "if I pay attention in class and do my homework then I will pass" into logic. State what each variable abbreviates.

(2) Translate $A \to (B \vee C)$ into English, making up a meaning for each of the variables.

(3) For this problem use the following abbreviations:

$$P = \text{"New York City has over 8 million inhabitants"}$$
$$Q = \text{"New York City has the best pizza in the world"}$$
$$R = \text{"Every New Yorker wishes they lived in Boston"}$$

Translate the following into English:

$$\begin{aligned} P \wedge Q \wedge {\sim}R \\ {\sim}R \to (P \wedge {\sim}Q) \\ Q \vee {\sim}Q \\ R \leftrightarrow ({\sim}P \wedge {\sim}Q) \\ P \wedge (P \to Q) \wedge {\sim}Q \\ {\sim}P \wedge (Q \vee {\sim}R) \end{aligned}$$

(4) Translate the following into English, making up meanings for the variables:

$$(A \wedge B) \wedge C$$
$$A \wedge (B \wedge C)$$

Do your translations convince you that these two logical statements are equivalent? Why or why not?

(5) Translate the following into English, making up meanings for the variables:

$$(A \vee B) \vee C$$
$$A \vee (B \vee C)$$

Do your translations convince you that these two logical statements are equivalent? Why or why not?

(6) Translate the following into English, making up meanings for the variables:

$$A$$
$$\sim A$$
$$\sim\sim A$$
$$\sim\sim\sim A$$

Which, if any, of these statements do you think are equivalent? Why?

(7) Translate the following into English, making up meanings for the variables:

$$A \to B$$
$$\sim B \to \sim A$$

Do your translations convince you that these two logical statements are equivalent? Why or why not?

(8) Translate the following into English, making up meanings for the variables:

$$A \leftrightarrow B$$
$$(A \to B) \wedge (B \to A)$$

Do your translations convince you that these two logical statements are equivalent? Why or why not?

(9) Suppose you know that the two propositions "if it is snowing then I should wear a coat" and "it is snowing" are true. What, if anything, can you conclude from them? Explain.

(10) Suppose you know that the two propositions A and $A \to B$ are both true. What, if anything, can you conclude from them? Explain.