

MATH 013: 3/19 WORKSHEET PERCENTS

Percents are a way to talk about fractions of a whole.

Percents.

Percent literally means per one hundred (think cent as in centimeter). $X\%$ means X hundredths:

$$X\% = \frac{X}{100}$$

- (1) 50% is the same as $\frac{1}{2}$. For example, 50% of 30 is 15.
- (2) 12% of 50 can be calculated as

$$12\% \cdot 50 = \frac{12}{100} \cdot 50 = \frac{600}{100} = 6.$$

- (3) A useful fact is that $X\%$ of Y is the same as $Y\%$ of X . This is because

$$X\% \cdot Y = \frac{X}{100} \cdot Y = \frac{Y}{100} \cdot X = Y\% \cdot X.$$

- (4) For example, 12% of 50 is the same as 50% of 12, which is 6.

Percents as relative increases.

Percents are used to talk about how much something increased. For example, the value of an investment might increase by 3% . If it was originally worth 100, now it's worth $100 + 3\% \cdot 100 = 103$. In general, an increase by $X\%$ means multiplying the value by $(100 + X)\%$. Namely, 100% for the starting value plus the increase of $X\%$. Equivalently, this is adding $X\%$ of the value.

A decrease can be represented by a negative percent. For example, if the investment decreases in value by 4% then it's now worth $100 - 4\% \cdot 100 = 96$.

- (1) A car is traveling at 60 mph then increases its speed by 40% . What is its new speed?
- (2) If a value doubles then it increased by what percent?

Percentiles.

Percents can be used measure position in an order. In a data set consisting of lots of values, the X th percentile is the value at which $X\%$ of other values are less than it.

- (1) If your SAT score was the 57th percentile, that means you scored better than 57% of test-takers.
- (2) The median is the same thing as the 50th percentile.
- (3) If your income is higher than 75% of people you would be in the 75th percentile.

Quantiles.

More generally, you can break a data set up into any number of chunks, rather than 100 for percentiles. Here are some common choices.

- *Quartiles* break it into four pieces, with boundaries at, 25%, 50%, and 75%.
- *Quintiles* break it into five pieces, with boundaries at 20%, 40%, and so on.
- *Deciles* break it into ten pieces, with boundaries at 10%, 20%, and so on.

- In 2022, the quintiles for household income in the US had boundaries: \$30,000, \$58,020, \$94,000, 153,000. That means that 20% of households made 30 thousand or less, 20% made between 30 and 58 thousand, and so on up to 20% making more than 153 thousand.

PRACTICE PROBLEMS

- (1) What is 40% of 100?
- (2) What is 40% of 15?
- (3) What is 28% of 50?
- (4) What is 6% of 50?
- (5) Profits on a record are shared amongst a band in the following manner: the lead singer gets 25%, the guitarist gets 20%, the bassist gets 15%, the drummer gets 15%, the manager gets 15%, and the theremin player gets 10%. If the total profits are 12,000 dollars, what is each member's share?
- (6) A youtube video had 2 million views at the beginning of the year. If its number of views has since increased by 3%, how many views is it at now?
- (7) Each year the value of an investment increases by 10%, where the investment is originally worth 1,000 dollars. Find it's value after one year. After two years. After three years.
- (8) Explain why two increases by 10% is more than one increase by 20%.
- (9) Your car depreciates in value each year. If its value decreases by 5% each year estimate how long until it is worth half its original value.
- (10) You are playing a video game and have the choice between the following two sets of equipment. Set *A* contains a single increase to attack, of 32%. Set *B* contains a 20% increase and a 12% increase. Which set will give you the bigger attack boost?
- (11) A 1 pound potato is 99% water by mass. It is left out to dehydrate, until it is 98% water by mass. How much does it weigh after this point?
- (12) A state legislator bemoans the quality of K–12 education in the state, and demands that standards be improved until the majority of students exceed the 50th percentile in mathematics skills. Is this a reasonable demand? Why or why not?