MATH 013: 3/17 WORKSHEET
COMBINING SETS

Set is mathematician speak for a collection of objects. When dealing with multiple sets, you want to be able to combine them. There are multiple ways to do this.

Intersection

The *intersection* of two sets A and B, written $A \cap B$ is the set of everything in both A and B. In set-builder notation:

$$A \cap B = \{x : x \in A \text{ and } x \in B\}.$$

Note that intersection corresponds to the logical connective AND. To help remember the symbols, they point the same way.

Union

The *union* of two sets A and B, written $A \cup B$ is the set of everything in either A or B. In set-builder notation:

$$A \cup B = \{x : x \in A \text{ or } x \in B\}.$$

Note that intersection corresponds to the logical connective OR. To help remember the symbols, they point the same way.

Set difference

The *set difference* of two sets A and B, written $A \setminus B$ is the set of everything in A but not in B. In set-builder notation:

$$A \setminus B = \{x : x \in A \text{ and } x \notin B\}.$$

This almost but not quite corresponds to the logical connective NOT. The reason we don't want to look at $\{x : x \notin B\}$ is that it's way too large to be useful. For instance, if B is the set of natural numbers, then $\{x : x \notin B\}$ contains $\sqrt{2}$, the Eiffel Tower, you, the emotion of anger, and many more things.

For these exercises, let $A = \{1, 2, 4, 8\}$, $B = \{2, 4, 6, 8\}$, and $C = \{2, 3, 5, 7\}$.

(1) What is $A \cup B$?
(2) What is $A \cap B$?
(3) What is $A \setminus B$?
(4) What is $B \setminus A$?
(5) What is $A \cup B \cup C$?
(6) What is $A \cap B \cap C$?

Like with logical connectives, we can represent set operations with Venn diagrams.

(1) Draw a Venn diagram which represents $A \cap B$.
(2) Draw a Venn diagram which represents $A \cup B$.
(3) Draw a Venn diagram which represents $A \setminus B$.

Properties of set operations

Because these operations are defined using basic logical connectives, we can use what we learned about them to tell us how these operations work.

- Order doesn't matter for $\cap$. That is, $A \cup B = B \cup A$ and $A \cup (B \cup C) = (A \cup B) \cup C$. This is because order doesn't matter for $\vee$.
- Order doesn't matter for $\cap$. That is, $A \cap B = B \cap A$ and $A \cap (B \cap C) = (A \cap B) \cap C$. This is because order doesn't matter for $\wedge$.
- (De Morgan's Laws) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
- (De Morgan's Laws) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

(1) Draw Venn diagrams which represent the two sides of the De Morgan's Laws equations, and use that to convince yourself the two sides really are equal.

Disjoint sets

Two sets are *disjoint* if they have no elements in common. In symbols, A and B are disjoint if $A \cap B = \emptyset$.

Counting

If A and B are disjoint, then $|A \cup B| = |A| + |B|$. In general, whether or not they are disjoint,

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

For set difference,

$$|A \setminus B| = |A| - |A \cap B|.$$

For these exercises, let $A = \{1, 2, 4, 8\}$, $B = \{3, 5, 7, 9\}$, and $C = \{2, 3, 5, 7\}$.

(1) What is $|A \cap B|$?
(2) What is $|A \cup B|$?
(3) What is $|A \cap C|$?
(4) What is $|A \cup C|$?
(5) What is $|B \cap C|$?
(6) What is $|B \cup C|$?

Here are some more abstract questions about counting the number of elements of a set.

(1) Explain why $|A| \leq |A \cup B|$. Can they be equal?
(2) Explain why $|A| \geq |A \cap B|$. Can they be equal?
(3) Explain why $A \subseteq B$ if and only if $A \cap B = A$.