MATH 013: 2/17 WORKSHEET
FRACTIONS 2

Last time we talked about what fractions are and how that informed how we multiply and divide fractions. Today we'll talk about other arithmetic operations on fractions.

Adding fractions with a common denominator.

$$\frac{\text{numerator}}{\text{denominator}} \qquad \text{or} \qquad \text{numerator/denominator}$$

Remember that you can think of a fraction as counting pieces of a whole. The *denominator*, written on the bottom of the fraction, is how many pieces the whole was broken into. The *numerator*, written on the top, is how many pieces you have. This tells you how add or subtract fractions with a common denominator. Keep the denominator and add (or subtract) the numerators.

$$\frac{n}{d} + \frac{m}{d} = \frac{m+n}{d} \qquad \text{and} \qquad \frac{n}{d} - \frac{m}{d} = \frac{m-n}{d}$$

- If you have 5 quarters and I give you 3 more, you have $\frac{5}{4} + \frac{3}{4} = \frac{5+3}{4}$ dollars, because you have $5+3$ quarters.
- $\frac{45}{100} - \frac{35}{100} = \frac{45-35}{100}$ can be thought of as with pennies. If you have 45 pennies and I take away 35 more you have $45-35$ left over.
- $\frac{2}{7} + \frac{6}{7} = \frac{2+6}{7}$ beacuse if you have 2 sevenths and I give you 6 more then you have $2+6$ sevenths.

Adding fractions without a common denominator.

If two fractions don't have a common denominator, to add them you first need to rewrite them with a common denominator. To rewrite a fraction with a new denominator, you need to multiply the numerator and denominator by the same value. Always, the product of the two denominators is a common demoniator. Often you will notice a smaller common denominator, and you can use that instead.

- To add $\frac{2}{5}$ and $\frac{2}{3}$ you need a common denominator for 5 and 3. Multiply them to get $5 \cdot 3 = 15$ as a commond denominator, which is in fact the least common denominator. Then to add them, convert to 15 as the denominator then add:

$$\begin{aligned} \frac{2}{5} + \frac{2}{3} &= \frac{2}{5} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{5}{5} \\ &= \frac{6}{15} + \frac{10}{15} \\ &= \frac{16}{15} \end{aligned}$$

- To subtract $\frac{3}{4}$ and $\frac{1}{2}$ you could take $2 \cdot 4 = 8$ as the common denominator. But you might instead notice that 4 is a smaller common denominator, and use that instead. That way, only one fraction needs to be rewritten to a new denominator.

$$\begin{aligned}\frac{3}{4}-\frac{1}{2}&=\frac{3}{4}-\frac{1}{2}\cdot\frac{2}{2}\\&=\frac{3}{4}-\frac{2}{4}\\&=\frac{1}{4}\end{aligned}$$

- To add $\frac{1}{4}$ and $\frac{5}{6}$ you could take $4\cdot 6 = 24$ as the common denominator, but you might notice that 12 is a smaller common denominator.

$$\begin{aligned}\frac{1}{4}+\frac{5}{6}&=\frac{1}{4}\cdot\frac{3}{3}+\frac{5}{6}\cdot 22\\&=\frac{3}{12}+\frac{10}{12}\\&=\frac{13}{12}\end{aligned}$$

- To add $\frac{1}{x+1}$ and $\frac{1}{x-1}$ you need a common demoniator. The best you can do here is to multiply the two denominators to get the common denominator of $(x+1)(x-1)$:

$$\begin{aligned}\frac{1}{x+1}+\frac{1}{x-1}&=\frac{1}{x+1}\cdot\frac{x-1}{x-1}+\frac{1}{x-1}\cdot\frac{x+1}{x+1}\\&=\frac{(x-1)}{(x+1)(x-1)}+\frac{(x+1)}{(x+1)(x-1)}\\&=\frac{(x-1)+(x+1)}{(x+1)(x-1)}=\frac{2x}{(x+1)(x-1)}\end{aligned}$$

Comparing fractions.

If two fractions have the same denominator, you can compare them by comparing numerators. The larger numerator gives the larger fraction. If the fractions don't have the same denominator, to compare them you must first rewrite them with a common denominator.

- $\frac{5}{9} > \frac{2}{9}$ because $\frac{5}{>}2$.
- To compare $\frac{5}{9}$ and $\frac{2}{3}$ first rewrite to a common denominator. You can choose 9 as the common denominator, so you are comparing $\frac{5}{9}$ and $\frac{2}{3}\cdot\frac{3}{3}=\frac{6}{9}$. And now you can see $\frac{5}{9}<\frac{6}{9}$ because $5 < 6$.

Multiple operations.

If you combine multiple operations—adding, subtracting, multiplying, dividing—then you follow the order of operations, using the rules for how to add/subtract and multiply/divide fractions.

- To calculate $\frac{2}{3} \cdot \frac{5}{4} - \frac{1}{6}$ you first multiply then you subtract. Note that to subtract will require finding a common denominator.

$$\begin{aligned}\frac{2}{3} \cdot \frac{5}{4} - \frac{1}{6} &= \frac{10}{12} - \frac{1}{6} \\ &= \frac{10}{12} - \frac{2}{12} \\ &= \frac{8}{12}\end{aligned}$$

  You might have alterantively noticed you could simplify $\frac{10}{12} = \frac{5}{6}$ and get a common denominator that way.

Practice Problems

Perform the following calculations

(1) $\dfrac{47}{100} - \dfrac{23}{100}$

(2) $\dfrac{52}{100} + \dfrac{71}{100}$

(3) $\dfrac{31}{100} - \dfrac{87}{100}$

(4) $\dfrac{5}{3} + \dfrac{3}{5}$

(5) $\dfrac{7}{4} + \dfrac{5}{6}$

(6) $\dfrac{1}{2} + \dfrac{1}{4} + \dfrac{1}{8}$

(7) $\dfrac{x}{4} + \dfrac{2x}{7}$

(8) $\dfrac{3}{2} + \dfrac{5}{x}$

(9) $\dfrac{x+1}{3} + \dfrac{4}{x}$

(10) $\dfrac{2}{3} \cdot \dfrac{1}{5} + \dfrac{3}{2} \cdot \dfrac{5}{4}$

(11) $\left(\dfrac{5}{3} + \dfrac{9}{4}\right) \cdot \dfrac{3}{2}$

(12) $\dfrac{1}{x+1} \cdot \dfrac{2}{x} - \dfrac{x}{x+1} \cdot \dfrac{-1}{x+2}$