MATH 211: 4/4 WORKSHEET
CONVERGENCE TESTS III

Ratio test. Consider $\sum_{n=1}^{\infty} a_n$ and compute $R = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$.

- If $r < 1$ then the series converges absolutely.
- If $r > 1$ then the series diverges.
- If $r = 0$ then the test in inconclusive.

Root test. Consider $\sum_{n=1}^{\infty} a_n$ and compute $R = \lim_{n \to \infty} \sqrt[n]{|a_n|}$.

- If $r < 1$ then the series converges absolutely.
- If $r > 1$ then the series diverges.
- If $r = 0$ then the test in inconclusive.

For each series say whether it converges or diverges.

(1) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!}$

(2) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$, where x is a fixed number

(3) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$

(4) $\displaystyle\sum_{n=0}^{\infty} \frac{(-x)^n}{(2n+1)!}$, where x is a fixed number

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(7) $\displaystyle\sum_{n=1}^{\infty} \frac{(n^2+3)^n}{(2n^2-4)^n}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^n}$

(9) $\displaystyle\sum_{n=2}^{\infty} \frac{5^n}{6^n-2^n}$

Confirm that the $R = 1$ case of the ratio and root tests are inconclusive.

(1) Consider a p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ and compute the ratio test R.
(2) Explain why this shows the ratio test is inconclusive.
(3) Compute the root test R for a p-series.
(4) Explain why this shows the root test is inconclusive.