MATH 211: 4/25 WORKSHEET
POLAR CALCULUS

1. Graphing

Use a graphing calculator, such as the one on `desmos.com/calculator` to explore graphs of polar functions. (If you type "theta" desmos interprets it as the greek letter and knows to do polar graphs. If you use another graphing calculator you may have to do something different to get polar functions.)

(1) Graph $r = \sin(k\theta)$ and $r = \cos(k\theta)$ for various values of k. What do you observe? Can you explain your observations?
(2) Graph $r = \theta$, $r = \theta^2$, and $r = e^\theta$. If $f(x)$ is a function which always increases what does the graph of $r = f(\theta)$ look like?
(3) Look some more at $r = e^\theta$. Zoom in and out on the graph. What do you observe, and can you use the properties of the exponential function to explain your observations?

2. Slopes

In Section 7.8 we learned two formulas to determine the direction of a tangent line of a polar function. Use these formulas to answer the following.

(1) The graph $r = \sin(3\theta)$ crosses the origin multiple times. Determine the angles at which it crosses the origin.
(2) Do the same for $r = \cos(3\theta)$.
(3) Confirm that the tangent line to the unit circle $r = 1$ at angle θ is at a right angle to θ.
(4) The graph $r = \sin(3\theta)$ has three points a maximal distance from the origin. Determine the slope of the curve at each point.

3. Area

In section 7.9 we learned how to find areas enclosed by polar curves.

(1) Find the area enclosed by the spiral $r = \theta$, where $0 \le \theta \le 2\pi$.
(2) Find the area enclosed by one loop of $r = \sin(3\theta)$.
(3) Suppose you know that $0 \le f(\theta) \le g(\theta)$ when $\alpha \le \theta \le \beta$. Determine a formula for the area between the curves $f(\theta)$ and $g(\theta)$ between the angles $\theta = \alpha$ and $\theta = \beta$.