MATH 211: 4/2 WORKSHEET
CONVERGENCE TESTS II

An *alternating series* is one terms alternate between positive and negative. When talking about an abstract alternating series, it is convenient to write it as

$$\sum_{n=1}^{\infty}(-1)^n a_n \qquad \text{or} \qquad \sum_{n=1}^{\infty}(-1)^{n+1} a_n,$$

depending on whether the first term is positive or negative. That way, a_n is the absolute value of the n-th term and represents its magnitude, while the -1 part gives the sign.

Alternating series test.
Consider an alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$. Assume that
(1) The terms a_n are decreasing $a_1 > a_2 > \cdots > a_n > \cdots$ and
(2) $\lim_{n\to\infty} a_n = 0$.
Then the series converges. Indeed, the sum is always between any two consecutive partial sums.

A series $\sum_{n=1}^{\infty} a_n$ is called *absolutely convergent* if $\sum_{n=1}^{\infty}|a_n|$ converges. If $\sum_{n=1}^{\infty} a_n$ converges but doesn't converge absolutely, we call it *conditionally convergent*.

Absolute convergence theorem.
Any absolutely convergent series converges. Thus, you can use the tests for positive-term series to check absolute convergence, which implies convergence.

For each series determine whether it converges absolutely, converges conditionally, or diverges. Which test did you use? If more than one can be applied, try giving multiple explanations. Note that you might need to remember tests from Monday.

(1) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

(2) $\displaystyle\sum_{n=1}^{\infty} (-1)^n$

(3) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$

(4) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n$

(5) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{3^n}{n!}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n!}$

(7) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \cdot \frac{\pi^{2n}}{(2n)!}$

(8) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^2}$

(9) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-3} \cdot \frac{n^2 - 1}{n^4 + n}$