MATH 211: 3/7 WORKSHEET

Recall the process of partial fraction decomposition of a rational function

$$\frac{n(x)}{d(x)}.$$

(1) If the degree of $n(x) \geq$ the degree of $d(x)$ then first do long division to pull out the polynomial part.
(2) Factor $d(x)$ fully.
(3) Each term in the factored form of $d(x)$ gives one or more fractions on the other side.
- A linear term $(x + a)$ gives $\frac{A}{x+a}$.
- A quadratic term $(x^2 + bx + c)$ gives $\frac{Ax+B}{x^2+bx+c}$.
- A term to power $k > 1$ gives $k + 1$ many fractions, from power k down to 1.
- Then work backward to solve for the parameters $A, B, \ldots$ in the numerators. With n parameters you will get a system of n linear equations to solve.

Here's some useful integrals to remember for integrating after you do the partial fraction decomposition.

$$\int \frac{\mathrm{d}x}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$
$$\int \frac{\mathrm{d}x}{x^2+b^2} = \frac{1}{b}\arctan\left(\frac{x}{b}\right) + C$$

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(1) Rewrite $\frac{2x-1}{(x-1)(x+2)}$ as a sum of two simpler fractions.
(2) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{2x-1}{(x-1)(x+2)}\,\mathrm{d}x.$$

(3) Use partial fraction decomposition to compute

$$\int \frac{x^2+4}{x(x+1)(x-1)}\,\mathrm{d}x.$$

(4) Use partial fraction decomposition to compute

$$\int \frac{3}{x^2-2}\,\mathrm{d}x.$$

Here's more integrals using partial fraction decomposition, with the extra complications we discussed.

(1) Rewrite $\frac{3x+1}{(x+3)^2}$ as a sum of two simpler fractions.

(2) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{3x+1}{(x+3)^2}\, \mathrm{d}x.$$

(3) Rewrite $\frac{1}{x^3+2x}$ as a sum of two simpler fractions.

(4) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{1}{x^3+2x}\, \mathrm{d}x.$$

(5) Rewrite $\frac{3x^2-4}{(x^2+1)^2}$ as a sum of two simpler fractions.

(6) Integrate

$$\int \frac{2x-1}{x(x^2+4x+4)}\, \mathrm{d}x.$$

(a) Do partial fraction decomposition to rewrite the fraction as a sum of two simpler fractions.

(b) One of these has denominator x, so is starightforward to handle.

(c) The other has denominator x^2+4x+4, and we don't have a rule to directly handle it. Instead, complete the square to rewrite the denominator in the form $(x+h)^2+k$.

(d) Then to integrate it you want to use the substitution $u = x+h,\ \mathrm{d}u = \mathrm{d}x$ so that the denominator looks like u^2+k.

(e) Now you can compute the integral like with earlier ones with denominator x^2+b^2.