MATH 211: 2/24 WORKSHEET

(1) We saw that the area between $1/x$, $x \geq 1$ and the x-axis is infinite. Rotate this curve around the x-axis and compute the volume of the resulting solid.

(2) Again think about $1/x$, $x \geq 1$ rotated around the x-axis. Calculating its surface area directly is hard, since you would have to integrate

$$\int_0^\infty \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}\,\mathrm{d}x.$$

Explain why this integral gives the surface area. In lieu of calculating it directly, instead compute a lower bound for the integral. It'll turn out that's good enough.

(a) First, explain why $\frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}} \geq \frac{2\pi}{x}$ for all $x \geq 1$. Then explain why that lets you conclude that

$$\int_0^\infty \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}\,\mathrm{d}x \geq \int_0^\infty \frac{2\pi}{x}\,\mathrm{d}x.$$

(b) Then compute $\int_0^\infty \frac{2\pi}{x}\,\mathrm{d}x$ to get a lower bound for the surface area.

(c) Compare the surface area of the solid to its volume. Think about what's going on. Whoa.

(3) For which exponents p is the integral

$$\int_1^\infty \frac{\mathrm{d}x}{x^p}$$

finite?

(4) For which exponents p is the integral

$$\int_0^1 \frac{\mathrm{d}x}{x^p}$$

finite?

(5) The *normal distribution* with mean μ ("mu") and standard deviation σ ("sigma") has the probability density function

$$\rho(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right),$$

where we write $\exp(u)$ for e^u to make it easier to read. Checking that this is a probability density function, i.e. that $\int_{-\infty}^\infty \rho(x)\,\mathrm{d}x = 1$, is hard. I won't ask you to do that, but think about why it is hard. Then set up and calculate an integral to confirm that the mean is μ. As a warmup, you might want to first consider the case where $\mu = 0$ and $\sigma = 1$.

(6) Let $f(x)$ be any function whose domain is $[0,\infty)$. Intuitively it should be clear that the arc length of $f(x)$ along $0 \leq x < \infty$ is infinite (why?). Justify this intuition using integration.