# MATH 211: 1/31 WORKSHEET

Recall the two parts of the fundamental theorem of calculus.

**FTC, part 1**
Suppose $f(t)$ is continuous on the interval $a \le t \le b$ and define

$$F(x) = \int_a^x f(t)\,\mathrm{d}t.$$

Then, $F'(x) = f(x)$ when $a < x < b$.

**FTC, part 2**
Suppose $f(x)$ is continuous on the interval $a \le x \le b$ and let $F(x)$ be any antiderivative of $f(x)$. Then,

$$\int_a^b f(x)\,\mathrm{d}x = F(b) - F(a).$$

differentiate the following functions.

- $a(x) = 8x^5 + 4x^3 - 6x$
- $b(x) = \sin(e^x) + e^{\sin x}$
- $c(x) = \dfrac{\tan x}{\arctan x}$
- $d(x) = x^{x^2+1}$
- $f(x) = \displaystyle\int_0^x e^{-t^2}\,\mathrm{d}t$

Calculate the following definite integrals.

(1) $\displaystyle\int_1^3 x^2 - 6\,\mathrm{d}x$

(2) $\displaystyle\int_0^{\ln \pi/2} e^x \sin(e^x)\,\mathrm{d}x$

(3) $\displaystyle\int_0^{\pi} \cos x\; e^{\sin x}\,\mathrm{d}x$

(4) $\displaystyle\int_0^{\ln(1+\pi^2)} g'(x)\,\mathrm{d}x$, where $g(x) = \cos(\sqrt{e^x - 1})$

Last semester you learned about using integration to find the area of a region bounded by curves. Namely, if a region is bounded above by a top curve $y = t(x)$, below by a bottom curve $y = b(x)$, to the left by a line $x = \ell$, and to the right by a line $x = r$, then the area of the region is

$$\int_\ell^r t(x)\,\mathrm{d}x - \int_\ell^r b(x)\,\mathrm{d}x = \int_\ell^r t(x) - b(x)\,\mathrm{d}x.$$

Graphically verify this fact.

(1) Draw a picture of a generic region bounded by $y = t(x)$, $y = b(x)$, $x = \ell$, and $x = r$.
(2) Show on your picture what regions have areas given by the integrals

$$\int_\ell^r t(x)\,\mathrm{d}x \qquad\qquad \text{and} \qquad\qquad \int_\ell^r b(x)\,\mathrm{d}x.$$

(3) Use this to explain why the formula for the area of the region works.

Now verify this fact by an alternate method.

(1) Draw a picture of a generic region bounded by $y = t(x)$, $y = b(x)$, $x = \ell$, and $x = r$.
(2) Divide this region up into infinitely thin vertical rectangles, each of width $\Delta x \approx 0$.
(3) Determine a formula for the area of each rectangle.
(4) Write a sum which represents the sum of the areas of the rectangle.
(5) Explain why this sum is a Riemann sum and identify the function this is a Riemann sum for.
(6) Write down the integral giving the value this Riemann sum is infinitely close to.
(7) Compare to the formula above for the area of the region, and explain your findings.

Test your knowledge by calculating the areas of the following regions. First sketch the region then calculate the area.

(1) The region bounded by the curves $y = e^x$, $y = -e^{-x}$, $x = 0$, and $x = 1$.
(2) The region bounded by the curves $x = e^y$, $x = -e^{-y}$, $y = 0$, and $y = 1$.
(3) The region bounded by the curves $y = 8 - x^2$ and $y = x^2$.