Study guide for Math 211 Midterm 2

April 9, 2025

Things to know

- How to do integration by parts.
- How to do integration by partial fraction decomposition.
- How to do integration by trig substitution.
- What convergence/divergence means for sequences and series, and how to compute the limit of a sequence.
- How to find the value of a geometric series.
- How to determine whether a series converges or diverges.
- How to find the radius of convergence of a power series.
- How to use a power series to compute derivatives and integrals.

You should expect some questions to be conceptual, asking you to explain something, while other questions are computational, asking you to calculate something.

The exam has nine questions.

Formula sheet

You will make and bring a formula sheet for the exam. It can be handwritten or typed, as you prefer. The only requirement is that it be a single 8.5 by 11 inch paper (front and back).

Here's some suggestions for what to put on your formula sheet:

- Formulas for integration.
- Any trig, etc. formulas you don't have memorized.
- Convergence tests for series.

Sample questions

1. Evaluate $\displaystyle\int x^2 e^{4x}\,\mathrm{d}x$.
2. Evaluate $\displaystyle\int \sin(x)\cdot\cos(2x)\,\mathrm{d}x$.
3. Evaluate $\displaystyle\int_0^{\pi/9} 3x\sin(3x)\,\mathrm{d}x$.

4. Evaluate $\int_0^{\pi/9} \ln x \, \mathrm{d}x$.

5. Evaluate $\int \cos^4 x \sin^3 x \, \mathrm{d}x$.

6. Evaluate $\int 3 \sin^2 x \, \mathrm{d}x$.

7. Evaluate $\int \sec^4 x \tan x \, \mathrm{d}x$.

8. Evaluate $\int \sin^2(x) \, \mathrm{d}x$.

9. Evaluate $\int_{-1}^{1} \sqrt{1-x^2} \, \mathrm{d}x$.

10. Evaluate $\int \frac{\mathrm{d}x}{(1+x^2)^3}$.

11. Evaluate $\int \sqrt{x^2-1} x^2 \, \mathrm{d}x$.

12. Determine the partial fraction decomposition of

$$\frac{1}{x^2+4x}.$$

13. Evaluate $\int \frac{1}{x^2+4x} \, \mathrm{d}x$.

14. Determine the partial fraction decomposition of

$$\frac{2x-1}{x(x^2+4)}.$$

15. Evaluate $\int \frac{2x-1}{x(x^2+4)} \, \mathrm{d}x$.

16. Determine the partial fraction decomposition of

$$\frac{x^2+x}{x(x-1)^2}.$$

17. Evaluate $\lim_{n\to\infty} (1+1/n)^n$

18. What is the exact value of $\sum_{n=0}^{\infty} \frac{3}{4^n}$?

19. What is the exact value of $\sum_{n=3}^{\infty} \frac{1}{10^n}$?

20. Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3}{n!}$$

21. Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3+4}{n^5-n}$$

22. Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{(2n)!}{n^n}$$

23. Does this series converge or diverge? Explain why

$$\sum_{n=0}^{\infty} \frac{n^3}{n!}$$

24. Does this series converge absolutely, converge conditionally, or diverge? Explain why

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2^n}$$

25. Does this series converge absolutely, converge conditionally, or diverge? Explain why

$$\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)}{n^2}$$

26. Determine the radius of convergence of $\sum_{n=2}^{\infty} x^n$.

27. You have a function $f(x)$ and you have computed that $f(2) = 1$, $f'(2) = 0$, $f''(2) = -1$, $f^{(3)}(2) = 3$, $f^{(4)}(2) = 1$. What is the degree 4 polynomial approximation to the Taylor series for $f(x)$ centered at $x = 2$?

28. Determine the Taylor series centered at $x = 1$ for e^x. Give your answer in sigma notation.

29. Determine the Maclaurin series for $\sin(x^3)$. Give your answer in sigma notation.