MATH 211: ORAL FINAL

Questions

For the oral final you will present your solutions to three of these questions. You will choose one question and I will choose the other two. Unless otherwise said, all answers to calculations should be exact answers.

(1) Evaluate
$$\int_0^1 \ln x \, \mathrm{d}x.$$
(2) Evaluate
$$\int u^3 \sin(u^2) \, \mathrm{d}u.$$
(3) Evaluate
$$\int_1^\infty \frac{4x^2+2}{x^4+x^2} \, \mathrm{d}x.$$
(4) State the test for when a p-series diverges or converges, and justify why the test holds.
(5) State both the "washer" and "cylindrical shell" methods for determining the volume of a solid and explain why the formulas work.
(6) Derive the arc length integration formulas for all three of the coordinate systems rectangular, polar, and parametric.
(7) The curve $y = x^{3/2}(1-x^2)^{1/4}$, from $x = 0$ to $x = 1$, is rotated around the x-axis to produce a solid. Determine the volume of this solid.
(8) The parametric equations $x(t) = \sqrt{44}t^2/2$ and $y(t) = \sqrt{5}t^3/3$ where $1 \leq t \leq 2$ define a curve in the plane. What is the length of this curve?
(9) The equation $r = \cos^2(\theta)$ in polar coordinates gives a lemniscate (∞ shaped curve). Determine the total area it encloses.
(10) Prove that if the decimal expansion of a number eventually repeats then the number is rational. That is, show that if the decimal expansion of a number x is eventually the digits $d_1 d_2 \cdots d_n$ repeating, then x can be written as a fraction of integers.
(11) Fix $\lambda > 0$. The function
$$\rho(n) = \frac{\lambda^n e^{-\lambda}}{n!}$$
gives the probability that a certain discrete random variable takes value n, where n ranges along the non-negative integers. Confirm that the total probability mass is 1 and compute the mean (= expected value) of the random variable.

Grading Rubric

Each question will be graded out of 33 points, plus 1 point for showing up, for 100 points total. Grades for each problem will be assigned according to the following rubric.

- **Flawless (33 points)** Student understood the problem completely and gave a full solution with no need for outside help. When asked to justify their process they gave a cogent explanation.
- **Near flawless (30 points)** Student was able to get through a complete solution and explanation, but made small errors and needed prompting to notice and correct them.
- **Adequate (25 points)** Student required minor outside help to know how to complete their solution, or was unable to fully explain their process.
- **Borderline (20 points)** Student required substantive outside help to get all the way through the problem. Student knew the key concepts, but may have struggled to explain how they apply to this problem.
- **Needs improvement (15 points)** Student did not know how to approach the problem without outside help. Student could not adequately explain their thought process.
- **Minimal engagement (10 points)** Student did not know how to approach the problem even with outside help, but recognized some of the concepts involved.
- **No response (0 points)** Student did not show up or said nothing related to the problem.

Formula sheet

$$\int \sin^n x \, \mathrm{d}x = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x \, \mathrm{d}x$$
$$\int \cos^n x \, \mathrm{d}x = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x \, \mathrm{d}x$$
$$\int \tan^n x \, \mathrm{d}x = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x \, \mathrm{d}x$$
$$\int \sec^n x \, \mathrm{d}x = \frac{1}{n-1} \sec^{n-1} x \sin x + \frac{n-2}{n-1} \int \sec^{n-2} x \, \mathrm{d}x$$

$$\mathrm{SA} = \int_a^b 2\pi f(x) \, \mathrm{d}s$$

$$\int \frac{\mathrm{d}x}{x^2 + a^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$$
$$\int \frac{\mathrm{d}x}{x + a} = \ln|x + a| + C$$

Detailed rules and format for oral final

You will schedule a 40 minute time slot for the oral final, which will be done at my office. Time slots are offered on the 5th, 6th, 7th, 9th, and 12th of May. You will sign up via a google sheet, which I will email to you. Please do not put off signing up for a slot; I don't want you to try to sign up last minute only to learn all the slots left conflict with your schedule.

You will present solutions to three of the questions. The first will be chosen by you, and the second and third by me. I will provide paper and a writing utensil. You will be graded according to the rubric in this document. After presenting your solutions I'll let you know your grade on the final and your overall grade in the class.

The rubric mentions outside help. Here's a few things that includes: consulting your notes; using a graphing calculator or computer algebra system (using a calculator for routine arithmetic is fine); asking me for help. The formula sheet in this document does not count as outside help, and I will have a copy available for you.

Advice for preparing

- Do all questions fully in advance! Practice writing down your solutions and explaining them.
- You are allowed to talk to classmates and tutors when preparing, but make sure you understand everything yourself. If I ask you "why did you do X?" then "Kent said it's the right thing" isn't a good answer.
- Check your solutions. Computer tools, especially graphing calculators, are great for checking your work.
- You can stop by office hours to ask about the class material, but I will not give hints for the questions for the final.