Math 211 Midterm 2

Wednesday, April 16

Name: Answer Key

This is the second midterm. There are nine questions, for a total of 100 points.

No electronic devices are permitted. You are allowed a single page of notes, standard 8.5 by 11 inches size. Carefully read each question and understand what is being asked before you start to solve the problem. Please show all your work and circle or mark in some way your final answers.

Turn in your note sheet with your exam when finished.

3. (10 points) You are still tutoring for calculus II. A student asks for help in setting up integrals using trig substitution. For the following integral, use an appropriate trig substitution to transform the integral into the θ domain. Write the $d\theta$ integral but do not solve it.

$$\int x^3\sqrt{x^2+1}\,dx$$

Give a sentence explaining why you chose the substitution you did.

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\sqrt{x^2+1} = \sec\theta$

$$\text{integral} = \int \tan^3\theta \sec\theta \sec^2\theta\, d\theta$$

$$= \int \tan^3\theta \sec^3\theta\, d\theta$$

$$= \int (1-\sec^2\theta)\sec^3\theta \cdot \tan\theta\sec\theta\, d\theta$$

I chose this substitution because of the x^2+1 pattern. This looks like the $\tan^2\theta + 1 = \sec^2\theta$ version of the Pythagorean identity, so it will simplify the integrand. Indeed, the integral can be solved by the substitution $u = \sec\theta$, $du = \tan\theta\sec\theta\, d\theta$.

Computational questions

5. (15 points) Calculate

$$\int_0^{\pi} x \sin(2x)\, dx.$$

$u = x \qquad v = -\frac{1}{2}\cos(2x)$

$du = dx \qquad dv = \sin(2x)\,dx$

$$= \left[-\frac{x}{2}\cos(2x) + \int \frac{1}{2}\cos(2x)\,dx \right]_0^{\pi} = \left[-\frac{x}{2}\cos(2x) + \frac{1}{4}\sin(2x) \right]_0^{\pi}$$

$w = 2x$

$dw = 2\,d\theta$

$$= \left[-\frac{\pi}{2} \cdot 1 + 0 \right] - \left[0 + 0 \right] = -\frac{\pi}{2}$$

8. (15 points) You are studying an object which is sinking in the ocean. You measure that its depth is 5 feet, its velocity is 12 feet per minute, and its acceleration is -2 feet per minute^2. Write the degree two Taylor polynomial that approximates the function $d(t)$ which gives its depth as a function of time. Use this Taylor polynomial to approximate what the object's depth and velocity will be after 1 minute.

$d(0)=5$ $\quad a_0=5$

$d'(0)=12$ $\quad a_1=12$

$d''(0)=-2$ $\quad a_2=-\frac{2}{2}=-1$

$d_2(t)=5+12t-t^2$ $\qquad d_2'(t)=12-2t$

$d(1)\approx d_2(1)=16$ feet

$v(1)\approx d_2'(1)=10$ feet/min