MATH 113: 4/18 WORKSHEET
THE MODAL ONTOLOGICAL ARGUMENT

One place modal logic gets applied is within philosophy to clarify arguments about metaphysics. An example of this is with *ontological arguments*, a species of argument purporting to show the necessary existence of god. The first of this kind of argument was by Anselm in 1078. In the latter half of the twentieth century philosophers used modal logic to present formalized ontological arguments.

Consider the following modal ontological argument, put forth by Hartshorne in 1965.[1] This argument has two premises and each step follows from a previous step either by the rules of TFL or by a rule of modal logic.

Here, let G be the proposition "god exists".

(1) (Premise) $G \rightarrow \Box G$
(2) (Premise) $\Diamond G$
(3) $\neg \Box G \leftrightarrow \Diamond \neg G$
(4) $\Diamond \neg G \rightarrow \Box \Diamond \neg G$
(5) $\neg \Box G \rightarrow \Box \Diamond \neg G$
(6) $\Box G \vee \neg \Box G$
(7) $\Box G \vee \Box \Diamond \neg G$
(8) $\neg \Box G \rightarrow \neg G$
(9) $\Box \neg \Box G \rightarrow \Box \neg G$
(10) $\Box \Diamond \neg G \rightarrow \Box \neg \Box G$
(11) $\Box \Diamond \neg G \rightarrow \Box \neg G$
(12) $\Box G \vee \Box \neg G$
(13) $\Box \neg G \leftrightarrow \neg \Diamond G$
(14) $\Box G \vee \neg \Diamond G$
(15) $\therefore \Box G$

Do the following:
(1) For each step of the argument, identify what logical rule or fact justifies it. Is this a rule of TFL, or of modal logic? If it's modal logic, which system is it from?
(2) State in plain English the meaning of the two premises and the conclusion.
(3) Try to make a case for the argument: why should we believe the premises?
(4) Try to make a case against the argument: either attack one of the premises or explain why we should take issue with one of the logical principles used in the proof.

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[1]Charles Hartshorne, *Anselm's Discovery: A Re-Examination of the Ontological Proof for God's Existence.* Open Court, 1965.