MATH 113: 4/11 WORKSHEET

Our next topic is another expansion to the expressiveness of our logic. But rather than someone at the front of the room telling you what it means, I want you to come up with what you think it should mean.

Our starting point will be truth-functional logic (so no quantifiers!). We will expand it by introducing a concept, which will need a new symbol. Since it can have multiple interpretations, we will write it as a blank box: $\Box$.

For the first interpretation, interpret $\Box$ as "I know …". So $\Box P$ means "I know P". For example, you might express "I know it will either rain or snow tomorrow" as $\Box(R \vee S)$. Given this interpretation, give the meaning of each of the following principles. Say whether you think it should be true or false, and why.

- $\Box P \to P$
- $P \to \Box P$
- $\Box P \to \Box\Box P$
- $\Box\Box P \to \Box P$
- $[\Box(P \to Q) \wedge \Box P] \to \Box Q$

For the second interpretation, interpret $\Box$ as "I believe …". So $\Box P$ means "I believe P". For example, you might express "if something is true then I believe it" as $P \to \Box P$. Given this interpretation, give the meaning of each of the following same principles. Say whether you think it should be true or false, and why.

- $\Box P \to P$
- $P \to \Box P$
- $\Box P \to \Box\Box P$
- $\Box\Box P \to \Box P$
- $[\Box(P \to Q) \wedge \Box P] \to \Box Q$

For the third interpretation, interpret $\Box$ as "it is morally obligatory to …". So you might express "it is morally obligatory to not murder" as $\Box\neg M$. Given this interpretation, give the meaning of each of the following same principles. Say whether you think it should be true or false, and why.

- $\Box P \to P$
- $P \to \Box P$

We might want to have two new concepts at once. We can't use a blank box for both of them, so let's rotate one of them by 45°: $\diamond$.

Interpret $\Box$ as "it is morally obligatory to ..." and $\diamond$ as "it is morally permissible to ...". For example, you might express "it is not permissible to eat meat" as $\neg\diamond M$. Given this interpretation, give the meaning of each of the following same principles. Say whether you think it should be true or false, and why.

- $P \to \diamond P$
- $\diamond P \to P$
- $\Box P \to \diamond P$
- $\Box P \to \neg\diamond\neg P$
- $\diamond P \to \neg\Box\neg P$

Interpret $\diamond$ as "it is possible that ..." and $\Box$ as "it is necessarily true that ...". For example, you might express "$2+2$ is necessarily 4" as $\Box T$ and you might express "it is possible that the global economy collapses within a year" as $\diamond C$. Given this interpretation, give the meaning of each of the following same principles. Say whether you think it should be true or false, and why.

- $P \to \diamond P$
- $\diamond P \to P$
- $\Box P \to \diamond P$
- $\Box P \to \neg\diamond\neg P$
- $\diamond P \to \neg\Box\neg P$
- $\diamond P \to \diamond\diamond P$
- $\diamond\diamond P \to \diamond P$
- $\diamond\Box P \to \Box P$

There's different ways you might interpret "possible"—humanly possible, possible within the laws of physics, logically possible. Do your answers change based on which you chose?