MATH 113: 3/28 WORKSHEET

Semantic concepts[1]

Interpretations are first-order logic's counterparts to what rows in a truth table were for truth-functional logic. They represent a possible meaning for the language. With these in hand we can now talk about *semantic concepts* which are counterparts to what we saw with truth-functional logic.

Like with TFL, we use the symbol $\vDash$ to represent *semantic entailment*, the core concept upon which others are built.

Semantic entailment.
Suppose $P_1, \dots, P_n$, and C are sentences in FOL. We write

$$P_1, \dots, P_n \vDash C$$

to mean that any interpretation in which each P_i is true must also assign C is true.

With TFL, we had a straightforward procedure to check entailment: namely, fill out the truth tables. Here we don't have such a procedure, since there are infinitely many possible interpretations. Nonetheless, we still have a procedure to check $P_1, \dots, P_n \nvDash C$. Namely, to show this we need to come up with an interpretation in which each P_i is true yet C is false.

Validities.
A sentence A in first-order logic is a *validity* if $\vDash A$, i.e. A is true in every interpretation.

Contradictions.
A sentence A in first-order logic is a *contradiction* if $\vDash \neg A$, i.e. A is false in every interpretation.

Equivalence.
Two sentences A and B are *logically equivalent* if they are true in exactly the same interpretations, i.e. both $A \vDash B$ and $B \vDash A$.

These three properties all are $\vDash$ properties. To check them requires saying something about every possible interpretation. So it can be very difficult to check them in general. On the other, checking that they don't hold requires only coming up with a single interpretation.

[1]This material corresponds to chapter 32 of the textbook.

Satisfiability.
Sentences $A_1, \ldots, A_n$ are *jointly satisfiable* if some interpretation makes them all true. They are *jointly unsatisfiable* if there is no interpretation in which they are all true.

Checking satisfiability requires only coming up with a single interpretation, whereas checking unsatisfiability requires reasoning about all interpretations.

Validity.
An argument $P_1, \ldots, P_n, \therefore C$ is *valid* if $P_1, \ldots, P_n \vDash C$, i.e. if there is no interpretation in which each premise is true but the conclusion is false. Otherwise it is *invalid*.

Checking validity requires reasoning about all interpretations while checking invalidity only requires coming up with a single interpretation.