MATH 113: 3/14 WORKSHEET

Binary predicates.

Some predicates most fruitfully understood as being a relationship between multiple objects. Take the sentence "Heracles's mother is Alcmene". You could formalize this as $A(h)$, where $h =$ Heracles and $A(x) =$ "x's mother is Alcmene". But it's more natural to formalize it as a predicate of two objects, not one: $M(h, a)$, where $h =$ Heracles, $a =$ Alcmene, and $M(x, y) =$ "x's mother is y". When a predicate is about two objects we call it a *binary* predicate, and we call a predicate about one object *unary*.

Another example.

Our domain is the positive integers. Let $L(x, y)$ be the predicate x is less than or equal to y, also written $x \leq y$. Which of these statements are true?

(1) $L(3, 7)$
(2) $L(5, 3)$
(3) $L(2, 2)$
(4) $\forall x\ L(0, x)$
(5) $\forall x\ L(x, 0)$

n-ary predicates.

You can also have a predicate about more than two objects. For example, you might formalize "Heracles's parents are Alcmene and Zeus" as $P(h, a, z)$, with appropriate symbolization key.

(1) Write out this symbolization key.

Quantifier order

An important question is, how does the order of quantifiers affect the meaning of a sentence?

Work in the domain of all humans with the symbolization key $M(x, y)$ = "x's mother is y".

(1) Explain why $\forall x\ \exists y\ M(x, y)$ can be rendered in English as "everyone has a mother". Is this a true sentence?
(2) Render $\exists y\ \forall x\ M(x, y)$ into English. Is this a true sentence? What can you can conclude about quantifier order?
(3) How would you symbolize "some people have two mothers" in first-order logic?

Work in the domain of real numbers with the symbolization key $L(x, y)$ = "$x \leq y$".

(1) Render $\forall x\ \exists y\ L(x, y)$ into English. Is this a true sentence?
(2) Render $\exists y\ \forall x\ L(x, y)$ into English. Is this a true sentence?
(3) Render $\forall x\ \exists y\ L(y, x)$ into English. Is this a true sentence?
(4) Render $\exists y\ \forall x\ L(y, x)$ into English. Is this a true sentence?

In mathematics the pattern $\forall x \exists y\ P(x, y)$ is very common, expressing that every x has a y satisfying $P(x, y)$.

(1) Provide a symbolization key and symbolize in first-order logic the mathematical fact "every linear function has a root". What is the domain?
(2) Provide a symbolization key and symbolize in first-order logic the mathematical fact "every polynomial has a derivative". What is the domain?
(3) Provide a symbolization key and symbolize in first-order logic the mathematical fact "every pair of integers has a least common multiple". What is the domain?

Work in the domain consisting of all monkeys and all bananas and use the following symbolization key to symbolize these sentences.

$M(x)$	x is a monkey
$B(x)$	x is a banana
$E(x, y)$	x wants to eat y

(1) Every monkey wants to eat some banana.
(2) Some monkey wants to eat every banana.
(3) No monkey wants to eat another monkey.
(4) Every banana wants to eat every monkey.
(5) Some banana wants to eat itself.
(6) There's a banana that wants to eat every monkey who wants to eat it.
(7) If there is a banana which wants to eat every monkey then there is a monkey who doesn't want to eat any banana.
(8) For any monkey who wants to eat a banana there is another monkey who wants to eat the same banana.