MATH 113: 2/5 WORKSHEET

To specify what is a grammatically correct sentence of truth-functional logic we give rules. (Here, φ is the Greek letter phi and ψ is the GReek letter psi. They are meta-syntatic variables which are used to refer to sentences.)

The following are sentences of truth-functional logic.

(1) Every variable is a sentence.
(2) If φ is a sentence then so is $\neg\varphi$.
(3) If φ and ψ are sentences then so is $(\varphi \wedge \psi)$.
(4) If φ and ψ are sentences then so is $(\varphi \vee \psi)$.
(5) If φ and ψ are sentences then so is $(\varphi \to \psi)$.
(6) If φ and ψ are sentences then so is $(\varphi \iff \psi)$.

Nothing else is a sentence of truth-functional logic.

These are the official rules of grammar, but we will relax them for ease of readability. When the meaning is unambiguous we will drop parentheses, and only keep them in when they're necessary to give the meaning.

- We write $P \to \neg Q$ instead of $(P \to \neg Q)$.
- We write $P \wedge Q \wedge R$ instead of $((P \wedge Q) \wedge R)$ or $(P \wedge (Q \wedge R))$. As we'll see next week, the meaning is the same no matter where the parentheses are placed.
- But we write $P \wedge (Q \vee R)$, because $P \wedge Q \vee R$ is ambiguous.
- But we write $\neg(P \iff Q)$ because $\neg P \iff Q$ means $(\neg P \iff Q)$.

Think of order of operations in algebra. Parentheses are there to make it explicit what happens first.