

MATH 113: 2/14 WORKSHEET

Every sentence of truth-functional logic can coherently be either true or false. This is the assumption on which the formal system is based. But there are sentences in ordinary language where you can't do this. One interesting species of these are *paradoxes*, sentences which cannot be consistently assigned any truth value.

(One sometimes sees people talk about "paradoxes" that are just counter-intuitive facts. For example, the so-called birthday paradox is the fact that if you have 23 people in a room it's more likely than not that two of them share a birthday. There's nothing impossible going on here, it's just that this probability fact is surprising to a lot of people. What we are concerned with today is sentences where it's impossible to say it's either true or false, not that the answer is surprising.)

To help understand the limits of what we can express in TFL, let's analyze a few paradoxes.

Liar's paradox

"This sentence is false."

Why is this sentence a paradox?

Truth-teller's paradox?

Is "this sentence is true" a paradox?

Multiple liars

- "The next sentence is false."
- "The previous sentence is false."

Are these sentences true or false? Explore.

Sometimes it's convenient to use variables as names to represent sentences, rather than saying stuff like "the next sentence".

Even more liars

- S_1 : " S_2 is false."
- S_2 : " S_3 is false."
- S_3 : " S_1 is false."

Are these sentences true or false? Explore.

Yet more liars. Here n is a positive integer.

- S_1 : “ S_2 is false.”
- S_2 : “ S_3 is false.”
- ⋮
- S_{n-1} : “ S_n is false.”
- S_n : “ S_1 is false.”

Are these sentences true or false? Explore.

Yablo’s paradox. Here we have an infinite list of sentences

- S_1 : “Each S_i is false, for $i > 1$.”
- S_2 : “Each S_i is false, for $i > 2$.”
- ⋮
- S_n : “Each S_i is false, for $i > n$.”
- ⋮

What’s going on here?

Curry’s paradox

“If this sentence is true, then Germany borders China.”

Explain what’s going on.

Quine’s paradox

“yields falsehood when preceded by its quotation” yields falsehood when preceded by its quotation

Why is this sentence paradoxical?