MATH 113: 2/12 WORKSHEET

One use of truth tables is they let us analyze semantic concepts.

The first concepts we investigated was validity. We can phrase arguments in truth-functional logic, so that an argument is sentences of TFL as premises with a sentence of TFL as conclusion. As we discussed, an argument is *valid* if it's impossible for the conclusion to be false when every premise is true. Using truth tables, we can cast this as: an argument is valid if every row with a T for each premise has a T for the conclusion. Think of it this way: a row in a truth table represents a possible world (= a possible combination of truth values for the variables). So this is saying that in any possible world where each premise is true the conclusion is also true.

To succinctly write an argument we will write e.g. $P; P \to Q; \therefore Q$. That is, we separate sentences with semicolons and put $\therefore$ before the conclusion

Is the argument $P; P \to Q; \therefore Q$ valid?

P	Q	P	$P \to Q$	Q
T	T			
T	F			
F	T			
F	F			

Is the argument $\neg Q; P \to Q; \therefore \neg P$ valid?

P	Q	$\neg Q$	$P \to Q$	$\neg P$
T	T			
T	F			
F	T			
F	F			

Is the argument $P \to R; Q \to R; P \vee Q; \therefore R$ valid?

P	Q	R	$P \to R$	$Q \to R$	$P \vee Q$	R
T	T	T				
T	T	F				
T	F	T				
T	F	F				
F	T	T				
F	T	F				
F	F	T				
F	F	F				

This can also be described using the language of *entailment*. A list $P_1, \ldots, P_n$ of sentences *entail* a sentence C if whenever each P_i is true C must be true. Thus, an argument $P_1; \ldots; P_n; \therefore C$ is valid if and only if $P_1, \ldots, P_n$ entail C.

In terms of truth tables, $P_1, \ldots, P_n$ entail C if every row in which each P_i is true has C is true. You sometimes see logicians write $P_1, \ldots, P_n \vDash C$ to mean this. (The $\vDash$ symbol is called the double turnstile.)

Does $\neg P \to P$ entail P? Why or why not?

Show that $P \to Q$ entails $\neg Q \to \neg P$. What about vice versa?

Your friend insists that two sentences are equivalent if and only if they entail each other. Are they correct? Why or why not?

We are also interested in understanding sentences by themselves, not as part of an argument. The pattern of when a sentence is true gives us information. If every row of the truth table for P is true then we call P a *tautology*. If every row is false we call P a *contradiction*. If some rows are true and some rows are false we say P is *contingent*.

Which of these are tautologies? Contradictions? Contingent?

- $P \vee \neg P$
- $P \wedge \neg P$
- $P \to P$
- $P \to \neg P$
- $(P \to Q) \vee (Q \to P)$
- $(P \to Q) \to P$
- $P \to (Q \to P)$

On Monday we talked about *equivalence*—two sentences are equivalent if their truth tables have the same value in each row.

- Explain why all tautologies are equivalent to each other. How does this make you feel?
- Explain why all contradictions are equivalent to each other. How does this make you feel?

- Explain why any sentence entails a tautology.
- Which sentences entail a contradiction? Explain.

In ordinary English, "if X then Y" and "X entails Y" are synonymous. Explain the connection between their formal counterparts by showing that $X \vDash Y$ if and only if $X \to Y$ is a tautology.