MATH 113: 1/31 WORKSHEET

> A sentence A is a *consequence* of sentences $B_1, \ldots, B_n$ if there is no world where $B_1, \ldots, B_n$ are all true but A is false. An argument is *valid* if its conclusion is a consequence of its premises. An argument is *invalid* if there is a *counterexample*—a world in which the premises are all true but the conclusion is false.

   Clue, based on the 1985 film of the same name starring Tim Curry, is a boardgame where players collect clues to determine the details of a murder: the killer, the murder weapon, and the location. For the purposes of this exercise, we'll use a simplified form of the game. There are three suspects (**Miss Scarlet**, **Colonel Mustard**, and **Mrs. Peacock**), three possible weapons (the **candlestick**, the **knife**, and the **revolver**), and three possible locations (the **billiard room**, the **dining room**, and the **library**).

   In this context, a world consists of three relevent data: which of the three is the killer, which of the three is the weapon, and which of the three is the location. An argument is valid if whenever these data fit the premises the conclusion must hold, and is invalid if there is a combination of these data in which the premises are all true but the conclusion is false.

   On the next page are various arguments a Clue player might use to reason their way to victory. For each, either explain why the argument is valid or produce a counterexample which demonstrates it is invalid.

Argument A.

- The killer is not Miss Scarlet.
- The killer is not Mrs. Peacock.
- ∴ The killer is Colonel Mustard.

Argument B.

- The weapon is either the candlestick or the knife.
- The weapon is not the knife.
- ∴ The weapon is the candlestick.

Argument C.

- The killer is Mrs. Peacock.
- The weapon is the revolver.
- ∴ The location is the library.

Argument D.

- The killer used the revolver.
- Miss Scarlet would never touch a gun.
- ∴ The killer is not Miss Scarlet.

Argument E.

- The killer used the revolver.
- Colonel Mustard knows how to handle firearms.
- ∴ The killer is Colonel Mustard.

Argument F.

- The weapon is not the candlestick.
- If the weapon is the knife, then the murder happened in the dining room.
- The murder happened in the library.
- ∴ The weapon is the revolver.

Argument G.

- If the killer used the revolver then the murder took place in the billiard room.
- If the killer used the candlestick then the murder took place in the dining room.
- The murder weapon is the knife.
- ∴ The location is the library.

Argument H.

- Colonel Mustard is not the killer.
- If Miss Scarlet committed the murder she would have used the revolver.
- If Mrs. Peacock committed the murder she would have used the knife.
- ∴ The weapon is not the candlestick.

Argument I.

- If Miss Scarlet is the murderer then she did it in the library.
- If the murder happened in the library then the weapon was the revolver.
- The weapon is the candlestick.
- ∴ The killer is not Miss Scarlet.

Argument J.

- Either the killer used the revolver in the billiard room or the killer used the candlestick in the dining room.
- The murder weapon is the knife.
- ∴ The location is the library.