

Math 113: Logic

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 - Data within a database.
 - Our moral obligations.
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 - The relationship between mathematical principles.
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 - Our moral obligations.
- Major uses in philosophy, mathematics, computer science, and linguistics.
- In this class our main focus will be **formal logic**, also called **symbolic logic**, where we express things in a formal language.
- Akin to how arithmetic sentences express facts about quantity, we can express logical facts in this language.

Arguments

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 - It is Thursday.
 - \therefore The dining hall will have sushi.
- (\therefore means “therefore” and is used to mark the conclusion of the argument.)
- If my professor hated me they wouldn't make me do homework.
 - My professor doesn't hate me.
 - \therefore My professor won't make me do homework.

Expressing arguments in ordinary language

- If it is Thursday then the dining hall will have sushi.
- It is Thursday.
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If it is Thursday then the dining hall will have sushi. It is Thursday. Therefore the dining hall will have sushi.

It's Thursday, and the dining hall has sushi on Thursdays, so there will be sushi today.

The dining hall will have sushi because they have sushi on Thursdays and today is Thursday.

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In ordinary language we don't always put the conclusion at the end and we don't always separate things into different sentences.

- Conclusions are often indicated by words like so, thus, therefore, hence, accordingly.
- Premises are often indicated by words like because, since, given that.

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Sentences in logic

Not every English sentence makes sense to be part of an argument.

- What's your favorite novel?
- Don't sleep through your alarm.

We are interested in **declarative** sentences, those that state such and such is the case.

Think: does it make sense to respond to the sentence with “this is true (false)” or with “I (dis)agree”?

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Think: does it make sense to respond to the sentence with “this is true (false)” or with “I (dis)agree”?

Warning! This does not track the fact versus opinion distinction. We can use logic to analyze the consequences of “Ethel Cain's new album slaps” just as well as we can to analyze “a hydrogen atom has one proton”.

Validity

We want to distinguish between these two kinds of argument.

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We want arguments where the conclusion is a **consequence** of the premises. If you agree with the premises then you must agree with the conclusion.

We call such an argument **valid**.

On the other hand, an argument is **invalid** if it's possible for the premises to be true but the conclusion to be false.

Soundness

If a valid argument moreover has that all of its premises are all true then we call it **sound**.

- If a number is prime then its square root is irrational.
- 3 is prime.
- $\therefore \sqrt{3}$ is irrational.

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Checking soundness can be very hard, and it won't be our focus! We will say that is someone else's problem.

- There's either an even number or odd number of electrons in the universe.
- There is not an odd number of electrons.
- \therefore There is an even number of electrons.