

MATH 210
WEEK 1 WRITING ASSIGNMENT SOLUTION

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Problem (Page 42 #25). *Show that if $a < b$ then $a < \frac{a+b}{2} < b$.*

Solution. Multiplying an inequality by a positive number or adding the same quantity to both sides will preserve the inequality. Thus we get:

$$a < b \quad \Rightarrow \quad \frac{a}{2} < \frac{b}{2} \quad \Rightarrow \quad a = \frac{a}{2} + \frac{a}{2} < \frac{a}{2} + \frac{b}{2}.$$

By adding $\frac{b}{2}$ instead we similarly get $\frac{a+b}{2} < b$. Done. □

Problem (Page 42 #26). *Show that any open interval contains infinitely many points.*

Solution. It suffices to prove this for bounded intervals (a, b) , because any unbounded interval contains a bounded interval. So suppose $a < b$. Let $a_0 = a$. Then set a_1 to be the average of a_0 and b . By the previous exercise this will be in the interval (a, b) . Now repeat this process. At stage n you've already defined $a < a_n < b$. Set a_{n+1} to be the average of a_n and b . Then a_{n+1} is between a_n and b , whence it is between a and b .

Since we can continue this process indefinitely, we get that there are infinitely numbers between a and b , as desired. □