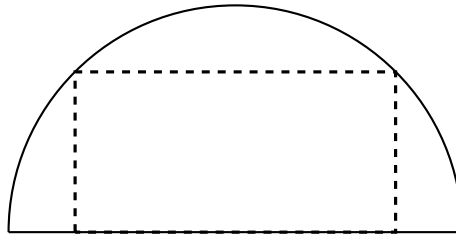


MATH 210: ORAL FINAL

QUESTIONS

For the oral final you will present your solutions to three of these questions. You will choose one question and I will choose the other two. Unless otherwise said, all answers to calculations should be exact answers.

- (1) Consider the curve $y = \sqrt[x]{\sin(\pi x/2)}$. Give an equation for the line tangent to this curve at the point $(1, 1)$.
- (2) Consider the curve given by the equation $\frac{\pi}{4}x^3 = \arctan(xy)$. What is the slope of the curve at the point $(1, 1)$?
- (3) Consider the function $f(x) = \ln(x^2 + 2x + 4)$. Find the x -coordinates of all local maximums and minimums of $f(x)$. Where is $f(x)$ increasing? Decreasing? What if you were asked about inflection points and concavity?
- (4) A rectangle is inscribed within the top half of a circle of radius 1 so that one side of the rectangle is along the diameter of the semicircle and the top corners of the rectangle touch the edge of the circle, as in the picture below. What is the maximum possible area for such a rectangle, and what are the side lengths which give that maximum area?



- (5) Determine the area of the region bounded by the curves $xy = -1$ and $y - 4x = 8$.

- (6) Calculate the following limits:

$$\lim_{n \rightarrow \infty} \frac{n^n}{n \ln n} \quad \text{and} \quad \lim_{n \rightarrow \infty} \frac{n^n}{n^2}$$

- (7) Your friend insists that the approximation of a definite integral given by the trapezoidal rule is the average of the approximations by the left and right Riemann sums. Explain why or why not your friend is correct. Additionally, your friend is approximating an integral of a decreasing function. For the three approximation methods mentioned—left Riemann sum, right Riemann sum, and trapezoidal rule—say why it overestimates the area, underestimates it, or it could go either way.
- (8) Calculate the following indefinite integral:

$$\int \frac{x + \arcsin x}{\sqrt{1 - x^2}} dx$$

- (9) Consider the function

$$f(x) = \begin{cases} \frac{\tan x}{x} - 1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Either calculate $f'(0)$ or explain why $f(x)$ is not differentiable at $x = 0$.

- (10) Using the rules for sin and cos and the combination rules as a starting point, explain why the differentiation rule for arctan works.
- (11) Newton's law of cooling says that the rate of change of the temperature of an object is proportion to the difference between its temperature and the ambient temperature. In symbols:

$$\frac{dT}{dt} = K(T - T_0),$$

where $\frac{dT}{dt}$ is the rate of change of the temperature T , K is a constant which depends upon the materials, and T_0 is the ambient temperature. (It is assumed that the ambient temperature T_0 is constant.)

You plunge a 300°C metal rod into a reservoir of water at 10°C . After 1 minute you measure that the temperature of the rod is now 200°C . Use Newton's law of cooling to estimate when the rod will be 20°C .

GRADING RUBRIC

Each question will be graded out of 33 points, plus 1 point for showing up, for 100 points total. Grades for each problem will be assigned according to the following rubric.

- **Flawless (33 points)** Student understood the problem completely and gave a full solution with no need for outside help. Perhaps a minor misstep was made along the way, but they were quick to correct it when prompted. When asked to justify their process they gave a cogent explanation.
- **Excellent (28 points)** Student was able to get through a complete solution, but required minor outside help or was unable to fully explain their process.
- **Adequate (23 points)** Student required substantive outside help either to start or to get all the way through the problem. Student knew the key concepts, but may have struggled to explain how they apply to this problem.
- **Needs improvement (18 points)** Student needed outside help and was nonetheless unable to complete the problem. Student could not adequately explain their thought process.
- **Minimal engagement (10 points)** Student did not know how to approach the problem even with outside help, but recognized some of the concepts involved.
- **No response (0 points)** Student did not show up or said nothing related to the problem.

DETAILED RULES AND FORMAT FOR ORAL FINAL

You will schedule a 30 minute timeslot for the oral final, which will be done at my office. Time slots are offered on the 9th, 10th, 11th, 16th, and 17th. You will sign up via a google sheet, which I will email to you. Please do not put off signing up for a slot; I don't want you to try to sign up last minute only to learn all the slots left conflict with your schedule.

You will present solutions to three of the questions. The first will be chosen by you, and the second and third by me. I will provide paper and a writing utensil. You will be graded according to the rubric in this document. After presenting your solutions I'll let you know your grade on the final and your overall grade in the class.

The rubric mentions outside help. Here's a few things that includes: consulting your notes; using a graphing calculator or computer algebra system (using a calculator for routine arithmetic is fine); asking me for help.

ADVICE FOR PREPARING

- Do all questions fully in advance! Practice writing down your solutions and explaining them.
- You are allowed to talk to classmates when preparing, but make sure you understand everything yourself. If I ask you "why did you do X ?" then "my classmate said it's the right thing" isn't a good answer.
- Check your solutions. Computer tools, especially graphing calculators, are great for checking your work.
- You can stop by office hours to ask about the class material, but I will not answer questions about nor give hints for the questions for the final.