Math 210 Midterm 2

Friday 8 November

Name: Answer Key

This is the midterm for unit 2.

Carefully read each question and understand what is being asked before you start to solve the problem. Please show your work in an orderly fashion, and circle or mark in some way your final answers.

No calculators nor other electronic devices are allowed.

1. (10 points) Compute the indefinite integral

$$\int 3\sqrt{t} - \frac{t}{2}\,\mathrm{dt}.$$

$$\int 3t^{1/2} - t/2\,dt = \frac{3t^{3/2}}{3/2} - \frac{t^{2}/2}{2} + C$$

$$= 2t^{3/2} + \frac{t^2}{4} + C$$

2. (10 points) On what domain is the following function continuous? Write your answer in interval notation

$$f(x) = \frac{x^2}{x^2 + 3x - 4}$$

where is $x^2+3x-4=0$

$(x+4)(x-1)=0$

$x=-4, 1$

Continuous on $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

3. (5 points) Compute the following limit:

$$\lim_{x\to 0} \frac{x^4 - 4x}{x^2 + 2x}. \quad \frac{0}{0}$$

$$\lim_{x\to 0} \frac{x(x^3-4)}{x(x+2)} = \lim_{x\to 0} \frac{x^3-4}{x+2} = \frac{-4}{2} = -2$$

4. (15 points) Compute the definite integral

$$\int_0^{\pi/2} 2\cos(x)e^{\sin(x)}\, dx.$$

$u = \sin x$

$du = \cos x\, dx$

$x = \pi/2 \to u = 1$

$x = 0 \to u = 0$

$$= \int_0^1 2e^u\, du$$

$$= 2e^u \Big|_0^1$$

$$= 2e^1 - 2e^0$$

$$= 2e - 2$$

5. (30 points) The questions across this page and the next are all about the function $a(x) = x^3 - 6x^2$.

(a) Compute $a'(x)$ and $a''(x)$.

$a'(x) = 3x^2 - 12x$

$a''(x) = 6x - 12$

(b) Solve for where $a'(x) = 0$ and $a''(x) = 0$, and use this to create sign diagrams for both $a'(x)$ and $a''(x)$.

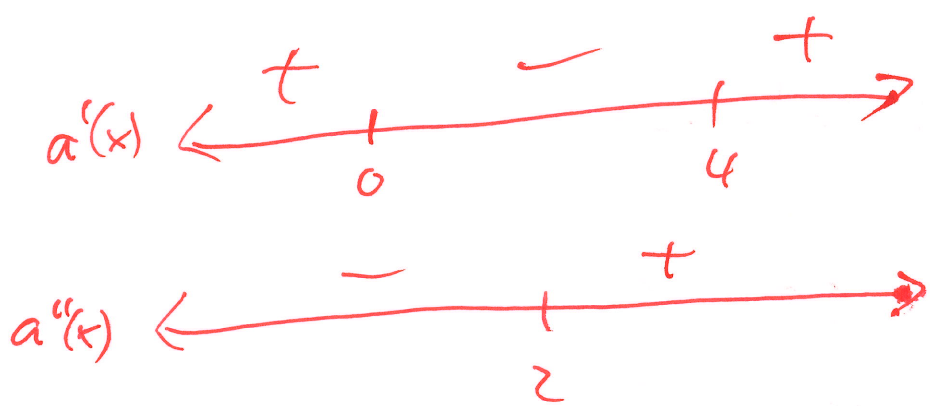
$3x^2 - 12x = 0$

$3x(x-4) = 0$

$x = 0, 4$

$6x - 12 = 0$

$x = 2$



$a'(-1) = 9$ pos

$a'(1) = -9$ neg

$a'(10) = 300 - 120$ pos

$a''(0) = -12$ neg

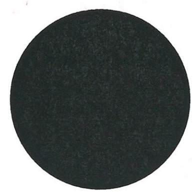
$a''(10) = 48$ pos

The questions across this page and the previous are all about the function $a(x) = x^3 - 6x^2$.

(c) Use the information you determined in part (b) to determine the following. Write "N/A" if it happens nowhere, and give appropriate answers in interval notation.

- $a(x)$ is increasing on $(-\infty, 0) \cup (4, \infty)$
- $a(x)$ is decreasing on $(0, 4)$
- $a(x)$ is concave up on $(2, \infty)$
- $a(x)$ is concave down on $(-\infty, 2)$
- $a(x)$ has a local maximum at $x =$ 6
- $a(x)$ has a local minimum at $x =$ 4
- $a(x)$ has an inflection point at $x =$ 2

6. (10 points) A featureless black circle has appeared in the center of the Simon's Rock campus. Although originally a mere dot, it has been steadily increasing in size, keeping a perfectly circular shape. You want to understand how quickly the circumference of the circle is changing, based on the presumption that its area is increasing at a constant rate. To this end, determine two things: (a) a formula for the circumfence c of the circle in terms of the area A; and (b) a formula for the rate of change c' of the circumference in terms of the area A and its rate of change A'.



Hint: Recall the formulas for circumference and area in terms of the radius r:

$$c = 2\pi r \qquad\qquad A = \pi r^2$$

$$\frac{A}{\pi} = r^2$$

$$r = \sqrt{\frac{A}{\pi}}$$ (only + b/c r can't be negative)

$$c = 2\pi \cdot \sqrt{\frac{A}{\pi}}$$

$$c = 2\sqrt{\pi} \cdot \sqrt{A}$$

$$c' = 2\sqrt{\pi} \cdot \frac{A'}{2\sqrt{A}}$$

$$c' = \sqrt{\pi} \cdot \frac{A'}{\sqrt{A}}$$

7. (10 points) You want to minimize the quantity

$$Q = \frac{x^2 - 2x}{y}$$

subject to the constraints $x \geq 1$, $y \geq 2$, and $2x + y = 12$. (a) Write an equation which describes Q as a function of a single variable. (b) Determine the domain of possible values for that single variable. (c) Without doing any further calculations, say whether a minimum must exist and explain why. **You do not have to differentiate your equation nor find the minimum value of Q.**

(a) $2x+y=12$

$y=12-2x$

$Q=\dfrac{x^2-2x}{12-2x}$

(b) $x \geq 1$

$y \geq 2 \Rightarrow 12-2x \geq 2$

$-2x \geq -10$

$x \leq 5$

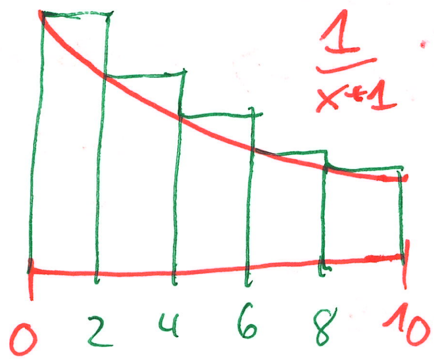
$1 \leq x \leq 5$

(c) $Q(x)$ is continuous on the closed bounded interval $[1,5]$. The extreme value theorem guarantees a ~~maximum~~ minimum exists.

8. (10 points) You are trying to approximate the definite integral

$$\int_0^{10} \frac{dx}{x+1}.$$

To that end you have enlisted the aid of an Elementary Functions student. Write a Riemann sum which approximates the integral with $n = 5$ rectangles, then write out the five terms of the sum, which you can then give to the ElFunc student to add up. **You do not have to simplify the terms nor compute the sum, but it should be written in a form so that someone with no knowledge of calculus nor sigma notation could compute it.**



$n=5 \Rightarrow \Delta x = 2$

$$\sum_{0}^{10} \frac{\Delta x}{x+1} = \frac{1}{0+1}\cdot 2 + \frac{1}{2+1}\cdot 2 + \frac{1}{4+1}\cdot 2 + \frac{1}{6+1}\cdot 2 + \frac{1}{8+1}\cdot 2$$

$$= \frac{2}{1} + \frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9}$$