MATH 211: 12-8 WORKSHEET

In lecture we saw formulas for area and arc length using polar coordinates.

- The area bounded by the rays $\theta = \alpha$ and $\theta = \beta$ and the curve $r = r(\theta)$ is given by

$$\int_\alpha^\beta \frac{1}{2} r(\theta)^2 \, \mathrm{d}\theta.$$

- The arc lenght of the curve $r = r(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is given by

$$\int_\alpha^\beta \sqrt{r(\theta)^2 + [r'(\theta)]^2} \, \mathrm{d}\theta.$$

(1) Check that a circle of radius a has area πa^2 by considering the polar function $r = a$ which describes the circle.
(2) Find the area enclosed by one petal of $r = \cos(3\theta)$. [If it's not clear what is meant by a petal, graph this polar function.]
(3) Work out a general formula for the area enclosed by one petal of $r = \cos(N\theta)$, where N is a positive integer.
(4) Set up an integral which gives the arc length of one petal of $r = \cos(3\theta)$.
(5) Find the arc length of the cardioid $r = 2 + 2\sin\theta$.
(6) Find the arc length of the spiral $r = \theta$ for θ from 0 to 2π.
(7) Find the arc length of the logarithmic spiral $r = e^\theta$ for θ from 0 to 2π.