MATH 211: 11-1 WORKSHEET

If the parametric equations $x(t)$, $y(t)$ describe the position of a particle in motion then the *vector* $\langle x'(t), y'(t)\rangle$ describes the velocity of the particle and its speed is $\sqrt{[x'(t)]^2 + [y'(t)]^2}$.

This can be used to get information about a parametrically defined curve:

- Slope at $(x(t), y(t)) = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y'(t)}{x'(t)}$.
- Arc length from $t = a$ to $t = b = \displaystyle\int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,\mathrm{d}t$.

Here are some exercises for thinking about calculus with parametrically defined curves.

(1) The equations $x(t) = t^3 - 7t$, $y(t) = (t+4)^2$, $t \geq 0$, describe the position of a particle moving over time. What is its speed at $t = 2$? What is the slope of the curve at $t = 2$?

(2) The equations $x(t) = 2t$, $y(t) = 5t + 3$, $-\infty < t < \infty$ describe a line. What is the slope of the line? Calculate the length of the line segment from $t = 0$ to $t = 4$ using an integral. Check that this matches the length you get if you use the pythagorean theorem directly.

(3) The equations $x(t) = e^t$, $y = e^{-2t}$ describe a curve. Calculate the arc length of the curve from $t = 0$ to $t = 1$.

(4) The equations $x(t) = t$ and $y(t) = t^{3/2}$, $0 \leq t \leq 4$ describe a curve. What is the length of this curve?

(5) Months ago you learned a formula for the arc length of a curve $y = f(x)$:

$$\text{arc length} = \int_a^b \sqrt{1 + [f'(x)]^2}\,\mathrm{d}x.$$

You could also describe this curve parametrically with $x(t) = t$ and $y(t) = f(t)$. Check that either way of describing the curve will give you the same formula for arc length.

(6) You can describe a curve in three dimensions with three equations for $x(t)$, $y(t)$, and $z(t)$, which you can think of as the curve traced out by a moving particle. How would you describe the velocity of the particle? How would you describe its speed? How would you calculate the arc length it traces out?