MATH 211: 11-3 WORKSHEET

- (Ratio test) Look at $r = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$.
  - If $0 \le r < 1$, then $\sum_{n=0}^{\infty} a_n$ converges absolutely.
  - If $r > 1$, then $\sum_{n=0}^{\infty} a_n$ diverges.
  - If $r = 1$, then the test is inconclusive.
- (Root test) Look at $r = \lim_{n\to\infty} \sqrt[n]{|a_n|}$.
  - If $0 \le r < 1$, then $\sum_{n=0}^{\infty} a_n$ converges absolutely.
  - If $r > 1$, then $\sum_{n=0}^{\infty} a_n$ diverges.
  - If $r = 1$, then the test is inconclusive.

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(1) Use the ratio test to check that $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges absolutely, where x is a fixed real number.

(2) Check whether $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ converges or diverges.

(3) Confirm that the $r = 1$ case of the ratio test is conclusive.

   (a) Show that for any p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ that $r = \lim_{n\to\infty} \frac{1/(n+1)^p}{1/n^p} = 1$.

   (b) Give a value for p for which the corresponding p-series converges, and give a value for which the p-series diverges.

(4) Use the root test to check whether $\sum_{n=0}^{\infty} \frac{(n^2+3)^n}{(2n^2-4)^n}$ converges.

(5) Does the series $\sum_{n=1}^{\infty} \frac{1}{n^n}$ converge or diverge? Why? Give two different explanations, one using the root test and one using another method.