MATH 211: 11-29 WORKSHEET

A differential equation is *separable* if it can be written in the form:

$$f(y)y' = g(x).$$

To solve a separable equation you can split $y' = \frac{\mathrm{d}y}{\mathrm{d}x}$ to the two sides then integrate:

$$\int f(y)\,\mathrm{d}y = \int g(x)\,\mathrm{d}x.$$

Taking the integrals on both sides gives an equation which describes the solution y.

(1) Solve the differential equation $y' = y$, finding the general solution.
(2) Solve the differential equation $y' = Ky$, where K is a fixed constant.
(3) Solve the differential equation $y' = 2xy^2$. Find the solution in explicit form.
(4) Solve the differential equation $y' = 2xy^2$ subject to the initial condition $y(0) = \frac{1}{4}$.
(5) Read through example 4.12 on page 385–386 of the textbook. Use the same technique to solve the next problem.
(6) You have an outdoor saltwater pool, as is normal in the Berkshires. The pool has a volume of 15,000 gallons. You want it to have the salinity of seawater, 35 parts per thousand of salt per water, which comes out to .3 lb of salt for each gallon of water. You miscalculated the size of your pool and put in 5500 lb of salt, which is too much. To fix this, you drain the pool at the rate of 5 gallons per minute while simultaneously filling it with fresh water at the rate of 5 gallons per minute. Meanwhile, you constantly stir the water to keep it the salt distribution uniform. Write a differential equation which describes the change in the quantity s of salt in the pool (in pounds) at time t (in minutes). Solve this differential equation to determine how long you need to drain the pool and refill it with fresh water to reach the optimal salt level of 4500 lb.