MATH 211: 10-18 AND 10-20 WORKSHEET

Here's some useful integrals to remember for partial fraction decomposition:

$$\int \frac{\mathrm{d}x}{ax+b} = \frac{1}{a}\ln|ax+b| + C$$
$$\int \frac{\mathrm{d}x}{x^2+b^2} = \frac{1}{b}\arctan\left(\frac{x}{b}\right) + C$$

(1) Rewrite $\frac{2x-1}{(x-1)(x+2)}$ as a sum of two simpler fractions.

(2) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{2x-1}{(x-1)(x+2)}\,\mathrm{d}x.$$

(3) Use partial fraction decomposition to compute

$$\int \frac{3x}{(x-3)(2x+1)}\,\mathrm{d}x.$$

(4) Use partial fraction decomposition to compute

$$\int \frac{x^2+4}{x(x+1)(x-1)}\,\mathrm{d}x.$$

Here's more integrals using partial fraction decomposition, with the extra complications we discussed.

(1) Rewrite $\frac{x^2+1}{(x+3)^2}$ as a sum of two simpler fractions.

(2) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{x^2+1}{(x+3)^2}\,\mathrm{d}x.$$

(3) Rewrite $\frac{1}{x^3+2x}$ as a sum of two simpler fractions.

(4) Use the partial fraction decomposition from the previous problem to compute

$$\int \frac{1}{x^3+2x}\,\mathrm{d}x.$$

(5) Rewrite $\frac{3x^2-4}{(x^2+1)^2}$ as a sum of two simpler fractions.

(6) Integrate

$$\int \frac{2x-1}{x(x^2+4x+4)}\,\mathrm{d}x.$$

   (a) Do the partial fraction decomposition to rewrite the fraction as a sum of two simpler fractions.
   (b) One of these has denominator x, so is starightforward to handle.
   (c) The other has denominator x^2+4x+4, and we don't have a rule to directly handle it. Instead, complete the square to rewrite the denominator in the form $(x+h)^2+k$.
   (d) Then to integrate it you want to use the substitution $u = x+h$, $\mathrm{d}u = \mathrm{d}x$ so that the denominator looks like u^2+k.
   (e) Now you can compute the integral like with earlier ones with denominator x^2+b^2.